德明政府中学
Dunman High School

TOPIC 3: DYNAMICS

(H1 and H2 2013)

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Content

- Newton's laws of motion
- Linear momentum and its conservation

Learning Outcomes

Candidates should be able to:

(a) state each of Newton's laws of motion.

(b) show an understanding that mass is the property of a body which resists change in motion.

(c) describe and use the concept of weight as the effect of a gravitational field on a mass.

(d) define linear momentum and impulse.

(e) define force as rate of change of momentum.

(f) recall and solve problems using the relationship $F = ma$, appreciating that force and acceleration are always in the same direction.

(g) state the principle of conservation of momentum.

(h) apply the principle of conservation of momentum to solve simple problems including elastic and inelastic interactions between two bodies in one dimension.

(i) recognize that, for a perfectly elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(j) show an understanding that, whilst momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Content Page

1. Introduction

Dynamics is the branch of mechanics where the forces that act on a body are not in equilibrium. The vector sum of the forces gives a resultant force that causes the body to accelerate. This resultant force causes change in motion. In Kinematics, we learned how a body would move under constant acceleration, if we combine the knowledge from Dynamics and Kinematics, we will be able to predict the motion of a body when we know the forces acting on the body. In the seventeenth century, Sir Isaac Newton formulated the three Newton's laws of motion and it is the basis behind Newtonian Mechanics. Today, Newtonian mechanics is useful for many engineering efforts in our everyday scale, like how an artillery shell travels in air, and it explains many phenomenon observed.

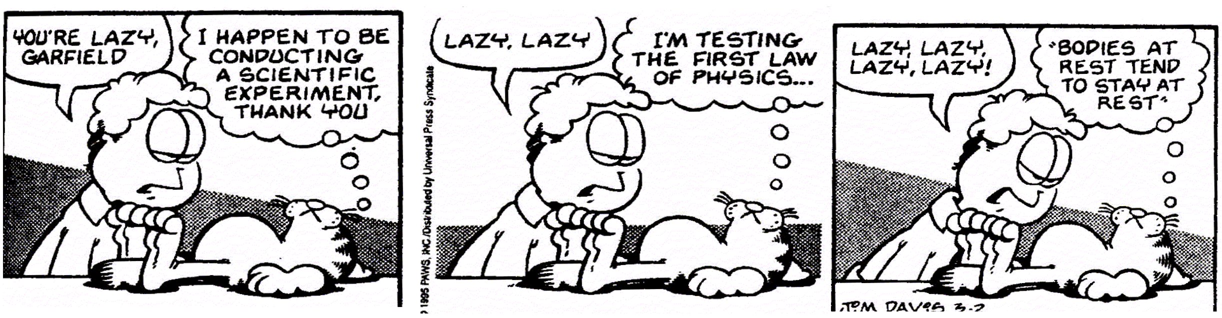
2. Newton's Laws of Motion

In this section, we will be learning Newton's laws from the basic principles using concepts of momentum and we will see how Newton's laws can be applied to everyday life.

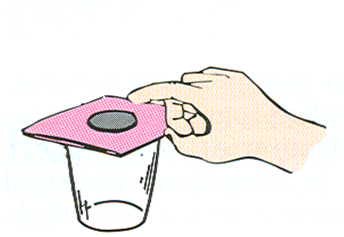
2.1 Newton's First Law of Motion (Law of Inertia)

Newton's First Law of Motion states that an object at rest will remain at rest and an object in motion will remain in motion at constant velocity in a straight line in the absence of an external resultant force.

It is also referred to as **Law of Inertia,** where inertia is the reluctance of a body to change its state of rest or uniform motion in a straight line.

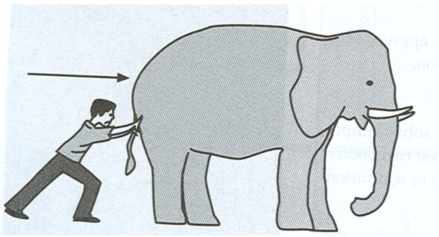


Food for thought

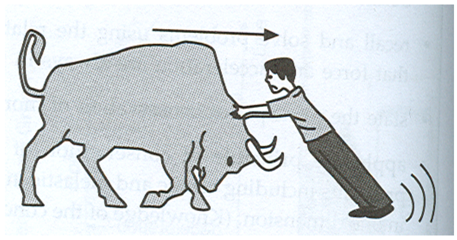


Can you think of a complete explanation for the tricks above?

The mass of a body is the amount of matter or substance in the body. It is a measure of its inertia. The bigger the mass, the bigger is the inertia of the body. Hence there is a great resistance to any change in velocity for a big mass.



Hard to start moving



Hard to stop moving once it is in motion

Newton's First Law of Motion can be used to explain the following observations:

Observation	Explanation
A passenger in a moving bus will 'jerk' forward when the bus driver suddenly brakes.	When the bus suddenly brakes, the passenger tends to keep moving at the previous speed and thus lurch forward.
A passenger in a stationary bus will 'jerk' backwards when bus driver suddenly accelerates.	When the bus suddenly accelerates, the passenger tends to stay at rest and thus lurch backwards.

Learning Objectives

(a) State each of Newton's laws of motion. [1st Law]

(b) Show an understanding that mass is the property of a body which resists change in motion.

2.2 Mass, Inertia and Weight

Mass is the amount of matter or substance in a body, which is a measure of the inertia of the body. It is a scalar quantity and is measured in kilogram (kg). The inertia of a body is the reluctance of the body to change its state of rest or uniform motion in a straight line, i.e. its reluctance to start moving, and its reluctance to stop once it has begun moving. The larger the mass, the larger is the inertia.

Weight of a body is defined as the *force* acting on it due to the effect of gravity.. Mathematically, for a body of mass m, its weight, $W = mg$, where g is the acceleration due to gravity. It is a vector quantity and is measured in newton (N).

Note that the *mass* of an object is constant all over the universe but its weight is a *force* whose magnitude depends on the value of g. An object's weight on Moon is only one-sixth of that on Earth, due to the Moon's weaker force of gravity (smaller g). However, it will be just as difficult to start to move a stationary object or stop it once it has begun moving on the Moon as on Earth. This is because *mass* (rather than *weight*) is a measure of the inertia of a body.

Lesson Objectives

(c) Describe and use the concept of weight as the effect of a gravitational field on a mass.

2.3 Momentum

Definition of Linear Momentum

Linear momentum of a body is defined as the product of mass of the body and its velocity, and it acts in the same direction as its velocity.

Mathematically, Momentum = mass x *velocity, or* $p = m\,v$ (Units: kg m s^{-1} or N s)

Momentum is a vector quantity. The direction of momentum is the same as the direction of the velocity. The unit of momentum is kg m s^{-1} or N s.

Momentum is the property of a body by virtue of its mass and velocity. While the velocity of a body indicates how fast it is moving, the momentum depends on both its velocity and mass.

Example 1

(a) Calculate the momentum of a 100 g bullet traveling at a speed of 400 m s^{-1} to the right.

(b) Calculate the velocity required for a running person of mass 60 kg to have the same momentum as the bullet.

Solution

(a) momentum of bullet = (0.100)(400) = 40.0 kg m s^{-1} to the right

(b) 40.0 = 60 v

v = 0.67 m s^{-1} to the right

Lesson Objectives

(d) Define linear momentum.

2.4 Newton's Second Law of Motion

Newton's Second Law of Motion states that the <u>rate</u> of change of momentum of an object is directly proportional to the <u>resultant</u> force and it <u>occurs in the direction</u> of the resultant force.

Note that you **CANNOT** state it as force is directly proportional to the rate of change of momentum.

Word/phrase	Meaning/Definition	Equation
change in momentum	Final momentum – initial momentum	$\Delta p = p_f - p_i$ $\Delta p = m\, v_f - m\, v_i$
rate of change of momentum	Derivative of change of momentum with respect to time or change in momentum divided by the time taken for the change (for constant force)	$\frac{dp}{dt}$ or $\frac{\Delta p}{\Delta t}$ (for constant force)

Mathematical Interpretation of Newton's Second Law of Motion:

The rate of change of momentum of a body, *p*, is directly proportional to the resultant force, F_{net}, acting on it and occurs in the direction of the resultant force.

$$F_{net} \propto \frac{dp}{dt} \qquad \Rightarrow \qquad F_{net} = k\frac{dp}{dt}$$

where k is the constant of proportionality.

When mass is constant, $F_{net} = k\frac{d(mv)}{dt} = km\frac{dv}{dt} = kma$ (where *a* is acceleration)

The unit of force is the newton (N or kg m s^{-2}).

One newton is defined as the force, which produces an acceleration of 1 m s^{-2} when it is applied to a mass of 1 kg.

Hence by definition, the constant k = 1.

$$F_{net} = kma$$

$$1\text{ N} = k\,(1\text{ kg})\,(1\text{ m s}^{-2})$$

$$k = 1$$

$$\therefore F_{net} = ma$$

where F_{net} is the resultant force in newton, *m* is the mass in kilogram and *a* is the acceleration in metre per second per second.

Note: The direction of the resultant force is the same as that of the acceleration of the body.

Learning Objectives

(a) State each of Newton's laws of motion. [2nd Law]

2.4.1 Free Body Diagram

An essential tool used to find the resultant or net force acting on a body is a free body diagram. A free body diagram shows all the forces acting on the body without showing the forces on the environment or other bodies. With this diagram, we can then set up equations to calculate the acceleration of the body.

(impt) The recommended procedure for analysis is as follows:

1. State clearly the body (or system) that is being considered and draw a simple sketch representing the body (or system).
2. Identify, mark and label on the sketch from part (1), all <u>external forces</u> acting on the body (or system), paying attention to the point of application of the forces.
3. Determine, if possible, the magnitude of the forces as identified in part (2).
4. Identify 2 suitable perpendicular axes (eg. x and y axis) for analysis (if required) and resolve all forces on the body (or system) along these 2 axes.
5. Decide for each of the 2 axes, the direction for which acceleration and force are to be assumed positive.
6. Apply Newton's law of motion individually to the free body for each of the chosen axes.

Note 1

The mass m to be used when applying Newton's 2nd Law in the form of $F_{net} = ma$, is the mass of the free body being considered.

Note 2

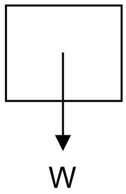
If a system is moving with velocity $\vec{v}$ and acceleration $\vec{a}$, all bodies in that system will also move with velocity $\vec{v}$ and acceleration $\vec{a}$.

When solving problems in Newtonian Mechanics, a free body diagram is useful to help you analyze the question effectively. Mastering the art of drawing a good diagram will lay the foundations for future topics such as Motion in a Circle, Forces etc.

2.4.2 Common Forces to Consider in Free-Body Diagrams

Weight, W

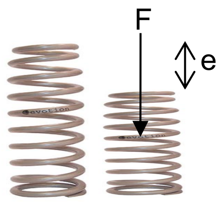
Gravitational force exerted by the source mass (i.e. Earth) on the body. Acts in the direction of the gravitational field strength, g, at the point of location of the body through the center of gravity of the body.



$$W = F_G = mg$$

Tensile and Compressive Forces within an Elastic Material

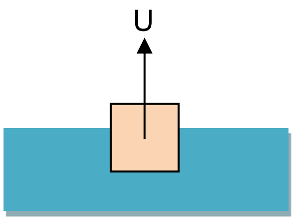
Force exerted by external bodies on the elastic material. It acts axially along the material. For an elastic material that obeys Hooke's Law, the extension/ compression *e* of the material is directly proportional to the tensile/ compressive force acting on it.



$$F = ke$$

Upthrust on a Body in a Fluid, U

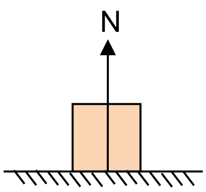
Force exerted by the surrounding fluid on the body. Acts vertically upwards through the centre of gravity of the fluid displaced by the body (centre of buoyancy). Archimedes' Principle - for a body placed in a fluid, the upthrust on the body by the fluid is equal in magnitude to the weight of the fluid displaced by body.



$$U = \rho_{fluid} V_{fluid\ displaced}\, g$$

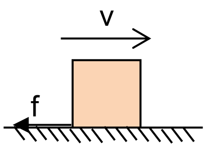
Normal Contact Force, N

Force exerted by the contact surface on the body. Acts in the direction of the normal to the point of contact.



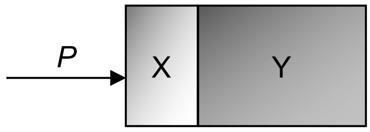
Frictional / Viscous Forces, f

Force exerted by the contact surface/ fluid on the body. It acts in the direction opposite to that of the motion of the body.



Example 2

Two blocks X and Y, of masses *m* and *3m* respectively, are accelerated along a smooth horizontal surface by a force *P* applied to block X as shown. Sketch the free body considering



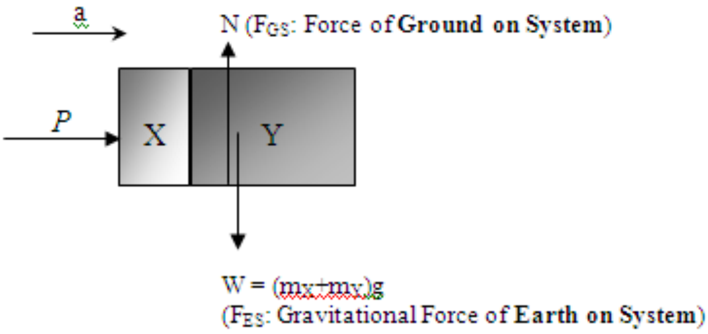
(i) both blocks as a system

(ii) block X alone and

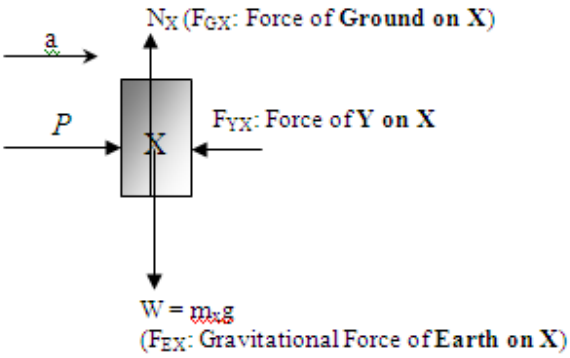
(iii) block Y alone

Solution

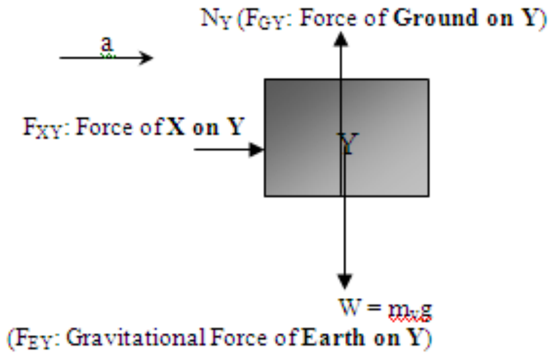
- Considering the free body of the system of the 2 blocks:



- Considering a free body of block X

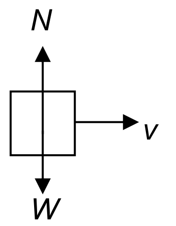
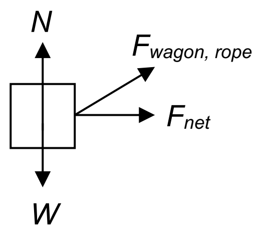
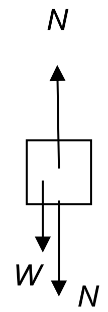


- Considering a free body of block Y



Example 3

What is wrong with the following free body diagrams?

Free Body Diagram		Mistake
A block sliding along a frictionless horizontal surface with a constant velocity.		Only <u>forces</u> are to be drawn on the free body diagram.
A wagon pulled along the floor by a rope held at an angle of 30° above the horizontal.		Resultant force should **<u>NOT</u>** be drawn on free body diagram.
A book resting on a table.		 should not be included as it acts on the table. Weight should act from the centre of the book

2.4.3 Constant Mass Problems

<u>Problems involving Case 1 (constant mass) can be solved by following six simple steps.</u>

Step 1: Draw the free body diagram

Step 2: Draw and label all the forces acting <u>on the body</u>.

Step 3: Assign positive direction (advised to assign in the direction of acceleration)

Step 4: Find the <u>net force</u> F in the direction of the acceleration.

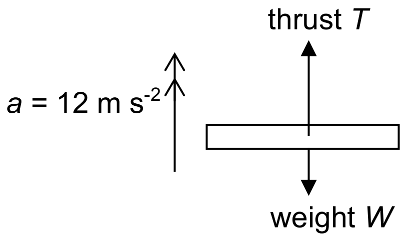
Step 5: Equate the net force to mass x acceleration. ($F = ma$)

Step 6: Solve the equation(s) to find the unknown(s).

Example 4

A model rocket, of mass 5.0 kg, rises with constant acceleration from rest to a height of 600 m in 10.0 s. Find the thrust that is exerted by the rotor blades during the ascent.

Solution



Using $s = ut + \frac{1}{2}at^2$

$600 = \frac{1}{2}a\,(10.0)^2$ or $a = 12.0$ m s^{-2}

Take upwards as positive,

$F_{net} = ma$

$T - W = ma$

$T - (5.0)(9.81) = (5.0)(12.0)$

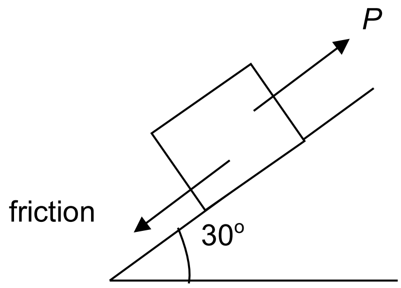
$T = 109$ N

Example 5

A car of mass 800 kg is moving up a hill inclined at 30° to the horizontal. The total frictional force on the car is 1000 N. Calculate the force P due to the engine on the car when the car is

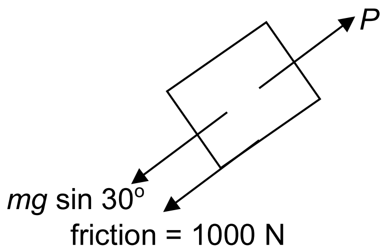
(i) accelerating up the plane at 2.0 m s^{-2}.

(ii) moving with a steady velocity of 15 m s^{-1}.



Solution

Consider forces on the car along the slope:



(i) + ve

$F_{net} = ma$

$P - mg \sin 30° - 1000 = 800\,(2.0)$

$P = 6.5 \times 10^3$ N.

(ii) + ve

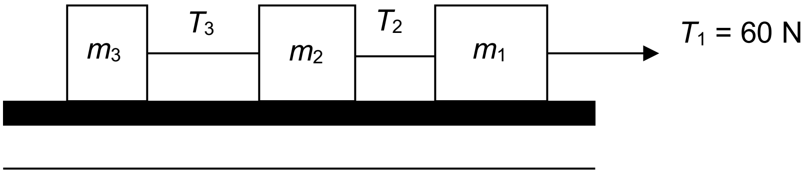
$F_{net} = ma$

$P - mg \sin 30° - 1000 = 0$

$P = 4.9 \times 10^3$ N.

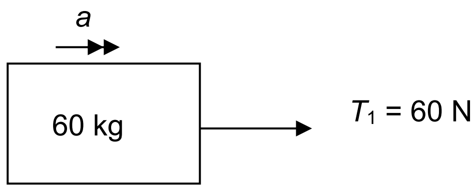
Example 6

Three blocks are connected on a horizontal frictionless table and pulled to the right with a force T_1 = 60 N. If m_1 = 30 kg, m_2 = 20 kg and m_3 = 10 kg, find the tensions T_2 and T_3.

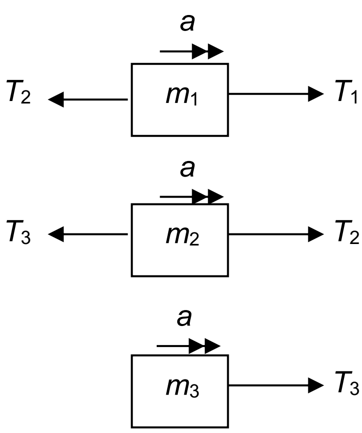


Solution

First, let us consider the whole system, i.e. consider m_1, m_2, and m_3 as one object.



acceleration of system $a = \frac{F}{m} = \frac{60}{60} = 1.0 \text{ m s}^{-2}$



Taking RHS as positive:

$T_1 - T_2 = m_1 a$ (1)
$= 30$

$T_2 - T_3 = m_2 a$ (2)
$= 20$

$T_3 = m_3 a$ (3)
$= 10$ N

Sub T_3 = 10 N into (2) gives

T_2 = 30 N

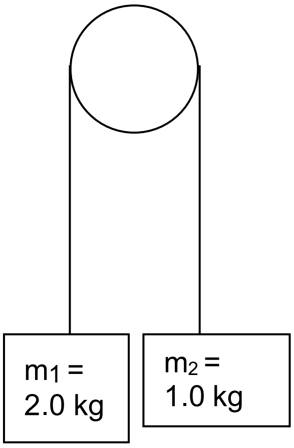
Alternatively, from (1), T_2 = 30 N

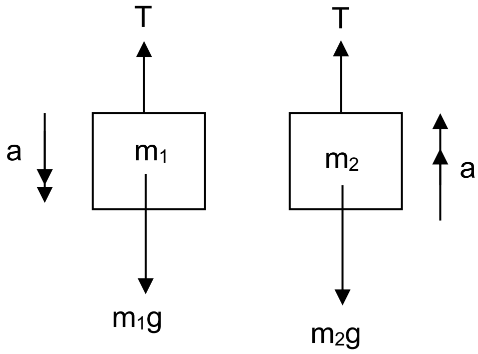
Note: For simplicity, only horizontal forces are shown.

Example 7

Two masses m_1 and m_2 are connected by a cord passing over a smooth pulley and held in the position shown. Find the tension in the cord and the acceleration of the two bodies when they are released from rest.

Solution





Consider mass 1, taking downwards positive

$m_1g - T = m_1a$ --- (1)

Consider mass 2, taking upwards positive

$T - m_2g = m_2a$ --- (2)

Equation (1) + (2),

$m_1g - m_2g = m_1a + m_2a$

$2.0g - 1.0g = (2.0 + 1.0)a$

$\therefore$ **a** $= g/3 = 3.3 \text{ m s}^{-2}$

Substitute *a* into (1)

$m_1g - T = m_1(g/3)$

$\therefore$ **T** $= 2m_1g/3$

$= 2 \times 2.0 \times 9.81/3 = 13$ N

Summary

(a) State each of Newton's laws of motion. [2nd Law]

(e) Define force as rate of change of momentum.

(f) Recall and solve problems using the relationship $F = ma$, appreciating that force and acceleration are always in the same direction.

2.5 Newton's Third Law of Motion

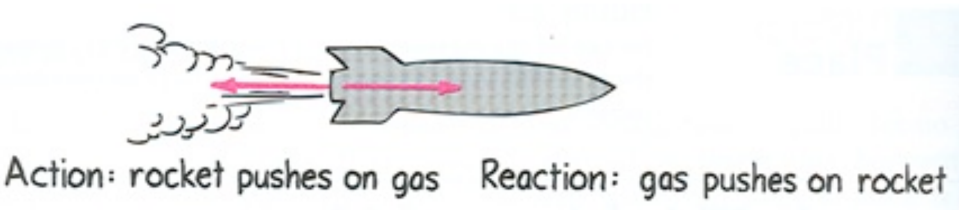
Definition of Newton's Third Law of Motion

If body A exerts a force on body B, then body B exerts a force of the same type that is equal in magnitude and opposite in direction on body A.

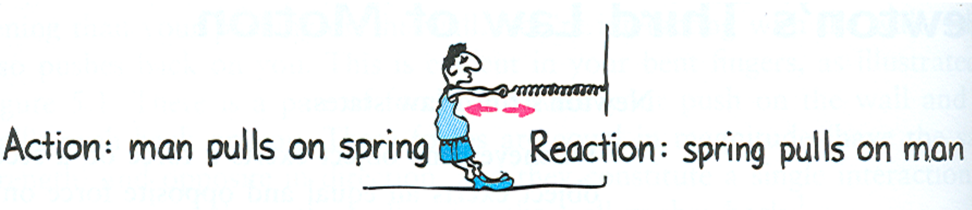
Note that you **CANNOT** state third law as action and reaction are equal and opposite.

Examples of action-reaction forces

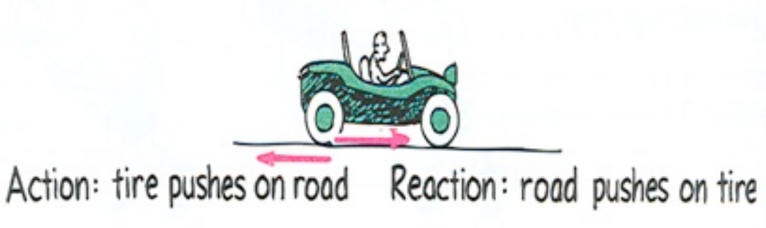
a)



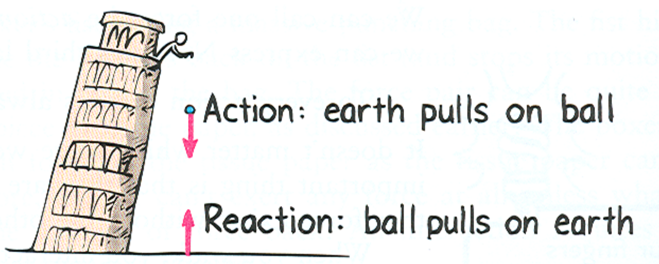
b)



c)



d)



Action-reaction forces:

(1) act on different bodies

(2) are equal in magnitude

(3) are opposite in direction

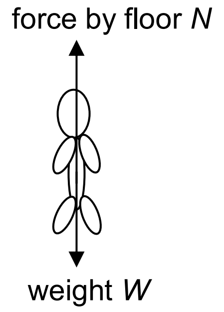
(4) are of the same kind (eg. If action is gravitational force, then reaction will be gravitational force too)

Example 8

Consider the two forces acting on the person who stands still – the downward pull of gravity and the upward support of the floor. Are these forces equal and opposite? Do they form an action-reaction pair? Why?

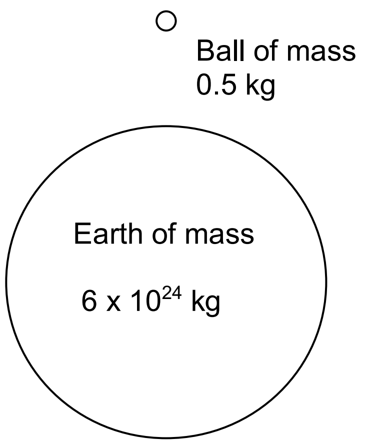
Solution

The forces are equal in magnitude and opposite in direction. However, they do not form an action-reaction pair as the forces act on the same body.



Example 9

Given that the mass of the ball in the picture is 0.5 kg, find the acceleration of the Earth towards the ball (assume the ball and Earth are the only two objects in the universe).



Solution

By Newton's 3rd Law,

|Force by Earth on ball| = |Force by ball on Earth|

$|F_{EB}| = |F_{BE}|$

$m_B\, a_B = m_E\, a_E$

$(0.5)(9.81) = (6 \times 10^{24})\, a_E$

$a_E = 8.2 \times 10^{-25}$ ms^{-2} (upwards)

Summary

(a) State each of Newton's laws of motion. [3rd Law]

(b) Recall and solve problems using the relationship $F = ma$, appreciating that force and acceleration are always in the same direction.

Example 10

A man of mass 80 kg stands on a platform scale in a lift as shown.

(a) Draw a free-body diagram to show the forces acting on the man

(b) Calculate the reading on the scale if the lift is

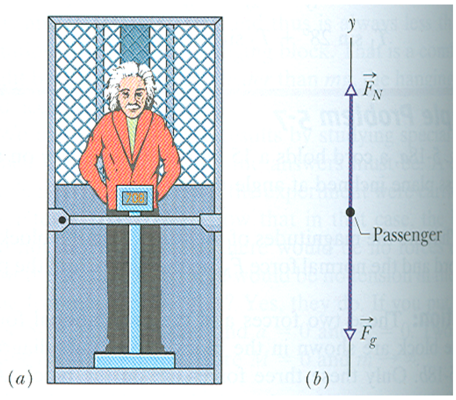
(i) stationary.

(ii) accelerating upwards at 3.0 m s^{-2}.

(iii) accelerating downwards at 3.0 m s^{-2}.

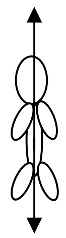
(iv) moving upwards at a constant speed of 10 m s^{-1}.

(v) accelerating downwards at 9.81 m s^{-2}.



Solution

(a) force by platform scale on man R



weight W

(b) (i) Lift is stationary:

$F = ma$

$R - W = ma$

$R - mg = m(0)$

$R = 780$ N

+ ve

(ii) $F = ma$

$R – W = ma$

$R = 1000$ N

+ ve

(iii) $F = ma$

$W - R = ma$

$R = 540$ N

+ ve

(iv) At constant speed, acceleration is zero.

$F = ma$

$R - W = ma$

$R - mg = m(0)$

$R = 780$ N

+ ve

(v) $F = ma$

$W - R = ma$

$mg - R = m(9.81)$

$R = 0$ N

+ ve

<u>Note</u>:

By Newton's 3rd Law, $|$force by man on platform scale$|$ = $|$force by platform scale on man$|$

Therefore reading on the scale = R.

From (b)(v), when $a = g$ (i.e. when the man accelerates downwards at the same rate as acceleration due to gravity) $R = 0$. The reading on the scale does not show any reading. This is the sensation of "apparent weightless".

3. Impulse and Momentum

In this section, we will go in depth into 2 new concepts, namely impulse and momentum. For the Newtonian dynamics we have learnt so far using concepts of velocity, acceleration and forces, we are more or less limited to constant net force and constant acceleration. In real world, we rarely encounter a constant net force and constant acceleration. The concepts of impulse and momentum will help us to understand real world phenomenon better as we can consider problems with non-constant net force acting on it.

3.1 Impulse

When we exert a force, *F*, on a body for a period, *t*, we will exert an impulse on the body proportional to the force exerted and the period of time where the force is exerted.

Definition of Impulse:

The impulse of a force is defined as the integral of a force over the time interval during which the force acts. Mathematically, the **impulse** of the force acting on the object is given by

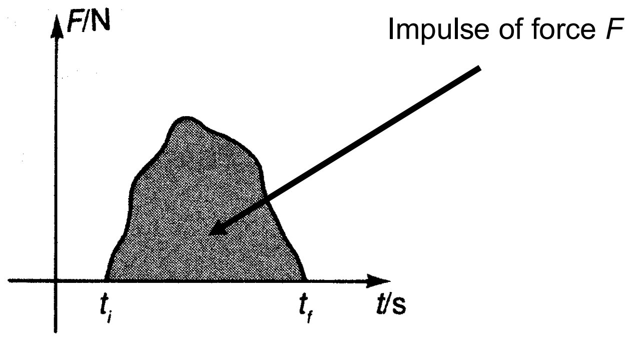
$$\int \boldsymbol{F}dt.$$

The unit of impulse is either N s or kg ms^{-1}.

It is easier to understand impulse graphically,

Impulse $= \int_{t_i}^{t_f} F dt$

= Area under *F-t* graph.



3.1.1 Impulse – Momentum Theorem

From Newton's Second Law of Motion,

$$F = \frac{dp}{dt} \Rightarrow Fdt = dp \Rightarrow \int_{t_i}^{t_f} Fdt = \int_{p_i}^{p_f} dp = p_f - p_i = \Delta p \quad .... \quad (1)$$

$$\int_{t_i}^{t_f} Fdt = \Delta p \qquad \text{------------------- Impulse-Momentum Theorem}$$

Impulse = Change in momentum

Impulse-Momentum Theorem: The impulse of a force acting on an object is equal to the change in momentum of the object.

The extent to which the momentum of a body is changed by a given net force depends on

(i) the magnitude of the net force

(ii) the duration in which the force acts on the body

Example 11

Explain why, when catching a fast moving ball, the hands are drawn back while the ball being brought to rest. Discuss whether your explanation has any bearing on the use of crushable boxes for packing eggs.

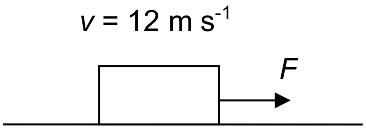
Solution

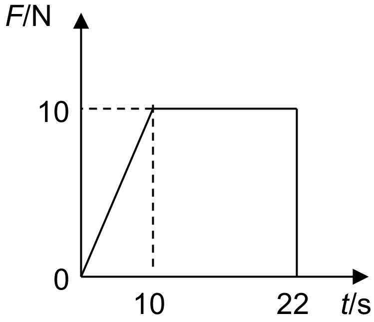
To stop a fast moving ball with a certain amount of momentum, we need to apply a force to change its momentum to zero. To prevent injury, the impulsive force needed to effect this change should be a small one. This can be achieved if the force is acting over a long time. By drawing the hands back, we are increasing the time of contact and hence reducing the average impulsive force acting on our hands. This will prevent serious injury.

The impulse of a force is defined as the integral of a force over the time interval during which the force acts. Impulse-Momentum Theorem states that the impulse of a force acting on an object is equal to the change in momentum of the object. The crushable boxes can compress hence it takes a longer time for the egg to come to a stop should it drops. Therefore the egg experiences a smaller force for the same change in momentum and will not break easily.

Example 12

An object, mass 10 kg, moving with an initial speed of 12 m s^{-1} has a force acted on it for 22 s as shown.





(a) Calculate the change in momentum after 25 s.

(b) What is the speed of the object at t = 25 s?

(c) Sketch the acceleration-time graph.

(d) Sketch the velocity-time graph.

(e) What is the average force on the object during the 22 s?

Solution

(a) From t = 0 s to t = 22 s, change in momentum = impulse of force = area under *F*-*t* graph

= area of trapezium

$= \frac{1}{2}(12+22)(10)$

= 170 kg m s^{-1}

From t = 22 to 25 s, there is no change in momentum. Thus the total change in momentum for 25 s is the same change in momentum for 22 s, which is equal to 170 kg m s^{-1}.

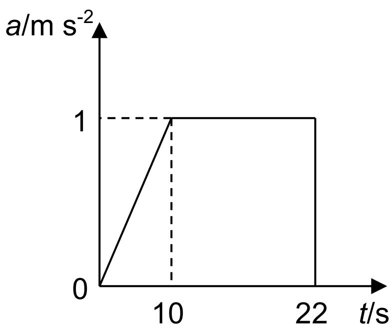
(b) change in momentum = mass x change in velocity

$$\Delta p = m(v_f - v_i)$$

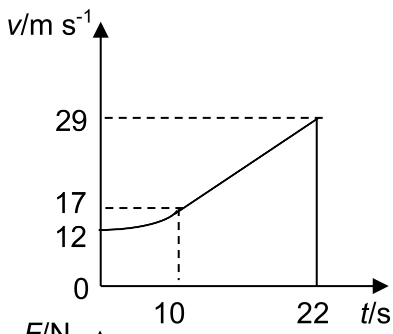
$$170 = 10(v_f - 12)$$

$$v_f = 29 \text{ m s}^{-1}$$

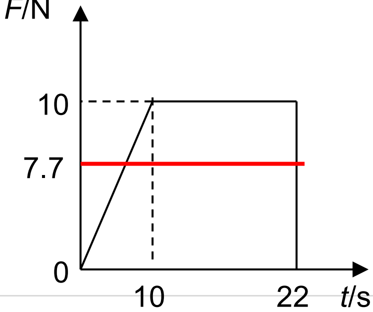
(c)



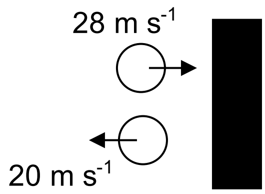
(d)



(e) $$< F > = \frac{\Delta p}{\Delta t} = \frac{\int F dt}{\Delta t} = \frac{\frac{1}{2}(22+12)(10)}{22} = \frac{170}{22} = 7.7N$$



Example 13



Consider a ball of mass 0.20 kg traveling with a velocity 28 m s^{-1} directly towards a wall. It hits the wall and bounces off in the opposite direction with a velocity of 20 m s^{-1}. Calculate the impulse due to the force by the wall.

Solution

Take RHS as positive,

impulse of force on ball = change in momentum of ball

$$= m(v_f - v_i)$$

$$= (0.20)(-20-28)$$

$$= -9.6 \text{ kg m s}^{-1}$$

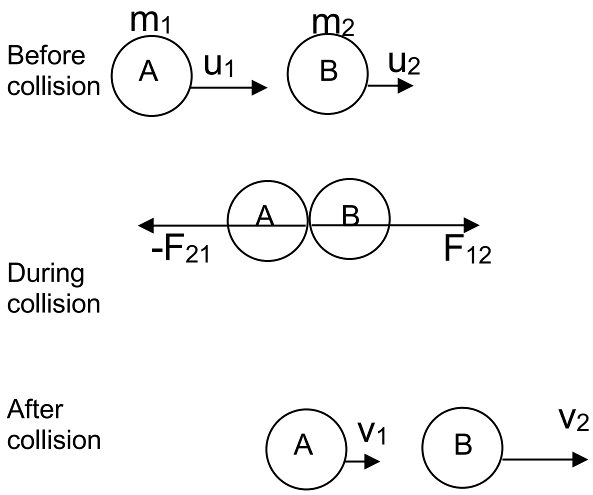
The impulse due to the force by the wall is 9.6 kg m s^{-1} to the left.

Summary

(d) Define impulse.

3.2 Conservation of Linear Momentum

Consider mass A traveling with velocity u_1 collides with mass B with velocity u_2. During the collision, the two masses are in contact and exert a force *F* on each other. The duration of impact is Δt.



By Newton's 2nd Law,

For A: $$F_{21} = \frac{\Delta\rho_1}{\Delta t}$$

$$= \frac{m_1v_1 - m_1u_1}{\Delta t}$$

For B: $$F_{12} = \frac{\Delta\rho_2}{\Delta t}$$

$$= \frac{m_2v_2 - m_2u_2}{\Delta t}$$

By Newton's 3rd Law, $F_{12} = -F_{21}$

$$\frac{m_2v_2 - m_2u_2}{\Delta t} = -\frac{m_1v_1 - m_1u_1}{\Delta t}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

<u>Total initial momentum = total final momentum</u>

Conservation of Linear Momentum

The principle of conservation of linear momentum states that the total momentum of a system **<u>remains constant</u>** provided <u>**NO** net external resultant force</u> acts on the system.

Example 14

A 7.0 kg bowling ball of velocity *u* collides head-on with a 2.0 kg bowling pin. The pin flies forward with a velocity of 3.0 m s^{-1} and the ball continues with a velocity of 1.8 m s^{-1}. What was the initial velocity *u* of the ball?

Solution

Taking $\rightarrow$ as +ve direction.

By Principle of Conservation of Momentum,

$m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$

$7.0\ u + 2.0x0 = 7.0x1.8 + 2.0x3.0$

$\therefore \quad u = 2.7\ m\ s^{-1}$

Example 15

A gun of mass 1.0 kg has a bullet of mass 0.1 kg inside. The bullet leaves the gun when fired at a velocity of 200 m s^{-1}. Calculate the velocity of the gun after firing.

Solution

Taking $\rightarrow$ as +ve direction.

By Principle of Conservation of Momentum,

$m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$

$1.0x0 + 0.1x0 = 1.0v_1 + 0.1 \times 200$

$\therefore \quad v_1 = -20\ m\ s^{-1}$

Note: Negative sign to indicate gun moves to the left

Types of Collisions

Elastic Collisions

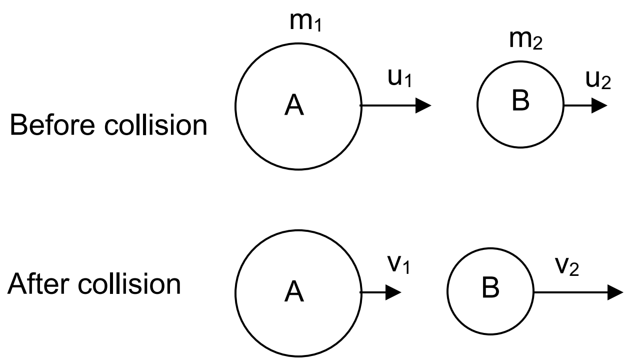
- **Total momentum is conserved.**
- **Total kinetic energy is conserved.**
- **Relative velocity of approach equals relative velocity of separation.**

Inelastic Collisions

- **Total momentum is conserved.**
- **Total kinetic energy is <u>not</u> conserved.**

3.2.1 Elastic Collisions

Consider two spheres A and B colliding elastically with each other. The initial velocities of A and B are u_1 and u_2 respectively. After the collision, A and B move off with v_1 and v_2 respectively.



Write down the equations relating to the conservation of momentum and conservation of kinetic energy.

$$\boxed{\text{Conservation of Momentum: } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2} \quad \text{---------} \quad (1)$$

$$\boxed{\text{Conservation of Kinetic Energy: } \tfrac{1}{2} m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2} \quad \text{----------} \quad (2)$$

Rewriting equation (2) gives:

$$m_1u_1^2 - m_1v_1^2 = m_2v_2^2 - m_2u_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad \text{----------} \quad (3)$$

Rewriting equation (1) gives:

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{----------} \quad (4)$$

Take eqn (3) ÷ (4) gives:

$$u_1 + v_1 = v_2 + u_2$$

$$u_1 - u_2 = v_2 - v_1 \quad \text{----------} \quad (5)$$

(Impt) For elastic collision, we can deduce that

Relative velocity of approach = Relative velocity of separation (in magnitude)

$$\vec{u}_1 - \vec{u}_2 = \vec{v}_2 - \vec{v}_1$$

From (5): $v_2 = u_1 - u_2 + v_1$ ---------- (6)

Sub (6) into (4): $m_1 (u_1 - v_1) = m_2 (u_1 - u_2 + v_1 - u_2)$

$m_1 u_1 - m_1 v_1 = m_2 u_1 - 2m_2 u_2 + m_2 v_1$

$(m_1 + m_2) v_1 = (m_1 - m_2) u_1 + 2m_2 u_2$

$$v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1 + \frac{2m_2}{(m_1 + m_2)} u_2 \qquad \text{----------} \quad (7)$$

Similarly, we can obtain:

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \qquad \text{----------} \quad (8)$$

Note: For elastic collisions where m_1, m_2, u_1 and u_2 are known, we can use the above equations (Eqns 7 and 8) to find v_1 and v_2. <u>However, it is advisable to use COM (Eqn 1) and COKE (Eqn 2) and the relative velocity equation (Eqn 5) to solve for the unknowns.</u>

Three Special Situations:

1. Two identical masses collide ie. $m_1 = m_2$, eg. two billiard balls collide with each other.

   $v_1 = u_2 \qquad v_2 = u_1$

   That means that the two masses exchange velocities after the collision.

2. A extremely big mass, m_1 collides with a very small mass at rest, m_2 ie. $m_1 >> m_2$, eg. bowling ball hit a stationary ping pong

   $v_1 \approx u_1 \qquad v_2 \approx 2u_1$

   The big mass m_1 continues with a speed close to its initial speed, while the small mass m_2 bounces off with a speed about twice the initial speed of m_1.

3. A very small mass m_1 collides with a very big mass at rest, m_2 , ie. $m_2 >> m_1$, eg. a ping pong striking a wall

   $v_1 \approx -u_1 \qquad v_2 \approx 0$

   The small mass m_1 rebounds with a speed close to its initial speed, and the big mass remains almost stationary.

Example 16

A 25.0 kg ball moving to the right at 20.0 m s^{-1} catches up and collides <u>elastically</u> with a 10.0 kg ball moving in the same direction at 15.0 m s^{-1}. Find the velocity of each object after the collision.

Solution

Conservation of Momentum: $m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$

Take RHS as positive, $25x20 + 10x15 = 25v_1 + 10v_2$

$650 = 25v_1 + 10v_2$

Relative velocity of approach = relative velocity of separation

$u_1 - u_2 = v_2 – v_1$

$20 - 15 = v_2 – v_1$

$5 = v_2 – v_1$

$v_1 = 17.1$ m s^{-1} $v_2 = 22.1$ m s^{-1}

Check: $\frac{1}{2} m_1u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ => KE is conserved.

Example 17

A 10.0 kg ball moving to the right at 10.0 m s^{-1} makes an <u>elastic</u> head-on collision with a 15.0 kg ball moving to the left at 30.0 m s^{-1}. Find the velocity of each object after the collision.

Solution

Conservation of Momentum: $m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$

Take RHS as positive, $10x10 + 15x(-30) = 10v_1 + 15v_2$

$-350 = 10v_1 + 15v_2$

Relative velocity of approach = relative velocity of separation

$u_1 - u_2 = v_2 – v_1$

$10 – (-30) = v_2 – v_1$

$40 = v_2 – v_1$

$v_1 = -38$ m s^{-1} (to the left) $v_2 = 2$ m s^{-1}

3.2.2 Inelastic Collisions

In most collisions, some changes in kinetic energy usually take place although the momentum of the system is always conserved. The loss in kinetic energy is due to the increase in internal energy and heat dissipated to the surroundings. In some cases, sound and light energy may be produced.

Inelastic Collision

- total momentum is conserved
- total kinetic energy is not conserved

Perfectly Inelastic Collision

- total momentum is conserved
- total kinetic energy is not conserved
- after collision, the 2 objects coalesce (move off together with a common velocity)

Example 18

A cue ball of mass 0.15 kg initial velocity 5.0 m s^{-1} collides with a billiard ball of mass 0.20 kg initially at rest. After the collision, the billiard ball moves with a velocity of 4.0 m s^{-1}.

(a) Calculate the velocity of the cue ball after the collision.

(b) Is the collision elastic?

Solution

(a) Conservation of Momentum: $m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$

Take RHS as positive, $0.15\text{x}5.0 + 0.2\text{x}0 = 0.15v_1 + 0.20\text{x}4.0$

$\therefore v_1 = -0.33 \text{ m s}^{-1}$

Note: The negative sign indicates that the cue ball is now traveling to the left.

(b) Total K.E. before collision = $\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_2u_2^2$

$= \frac{1}{2} \text{ x } 0.15 \text{ x } 5.0^2 + 0$

$= 1.9 \text{ J}$

Total K.E. after collision = $\frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$

$= \frac{1}{2} \text{ x}0.15\text{x}0.33^2 + \frac{1}{2} \text{ x}0.20\text{x}4.0^2$

$= 1.6 \text{ J}$

Since KE is lost during the collision, the collision is an inelastic collision. 0.3 J of energy may be converted to heat and sound energy.

Important!

The relative velocity equation $\mathbf{u_1 - u_2 = v_2 - v_1}$ **does not** apply for inelastic collisions.

Note

We may not be sure of the directions of the balls/objects after collision. In order to solve problems, we need to make an assumption of the direction. If we obtain a negative value, we will know that it actually moved in the opposite direction.

Example 19

A ball A of mass 0.10 kg moves with a velocity of 6.0 m s^{-1} undergoes perfectly inelastic collision with ball B of mass 0.20 kg initially at rest. Find the velocity of ball A after collision.

Solution

Note: For perfectly inelastic collision, $v_A = v_B = V$ = common velocity

Taking $\rightarrow$ as positive direction.

Conservation of Momentum:

$$m_Au_A + m_B u_B = m_Av_A + m_Bv_B$$
$$m_Au_A + m_B u_B = (m_A + m_B) V$$
$$0.10 \times 6.0 + 0 = 0.30 V$$
$$V = 2.0 \text{ m s}^{-1}$$
$$\therefore v_A = 2.0 \text{ m s}^{-1}$$

Summary

(f) state the principle of conservation of momentum.

(g) apply the principle of conservation of momentum to solve simple problems including elastic and inelastic interactions between two bodies in one dimension.

(h) recognize that, for a perfectly elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(i) show an understanding that, whilst momentum of a system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Dynamics Summary

N1L: If F = 0, v is 0 or constant.

N2L: For constant mass: **F = ma**

N3L: $\mathbf{F_{AB} = - F_{BA}}$ $\quad |F_{AB}| = |F_{BA}|$

Impulse = $\int F dt$ **=** area under the Force-time graph

For a constant force (From N2L)

$$F = \frac{\Delta mv}{\Delta t} \qquad F\Delta t = \Delta mv \qquad \textbf{Impulse} = F\Delta t = \Delta mv$$

Impulse = change of momentum

If the duration of the impulse is increased (e.g. by air bag), the average force will decrease for the same change in momentum.

	Elastic	**Inelastic**
COM	√	√
COKE	√	X
Equation	(1), (2) and (3)	(1) only

Conservation of Momentum: $m_1u_1 + m_2 u_2 = m_1v_1 + m_2v_2$ --------- (1)

Conservation of Kinetic Energy: $\frac{1}{2} m_1u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ --------- (2)

Relative velocity of approach =
Relative velocity of separation: $u_1 - u_2 = v_2 - v_1$ -------- (3)
(in magnitude)

Perfectly inelastic collision, the objects move off together after the collision, i.e. $v_1 = v_2$