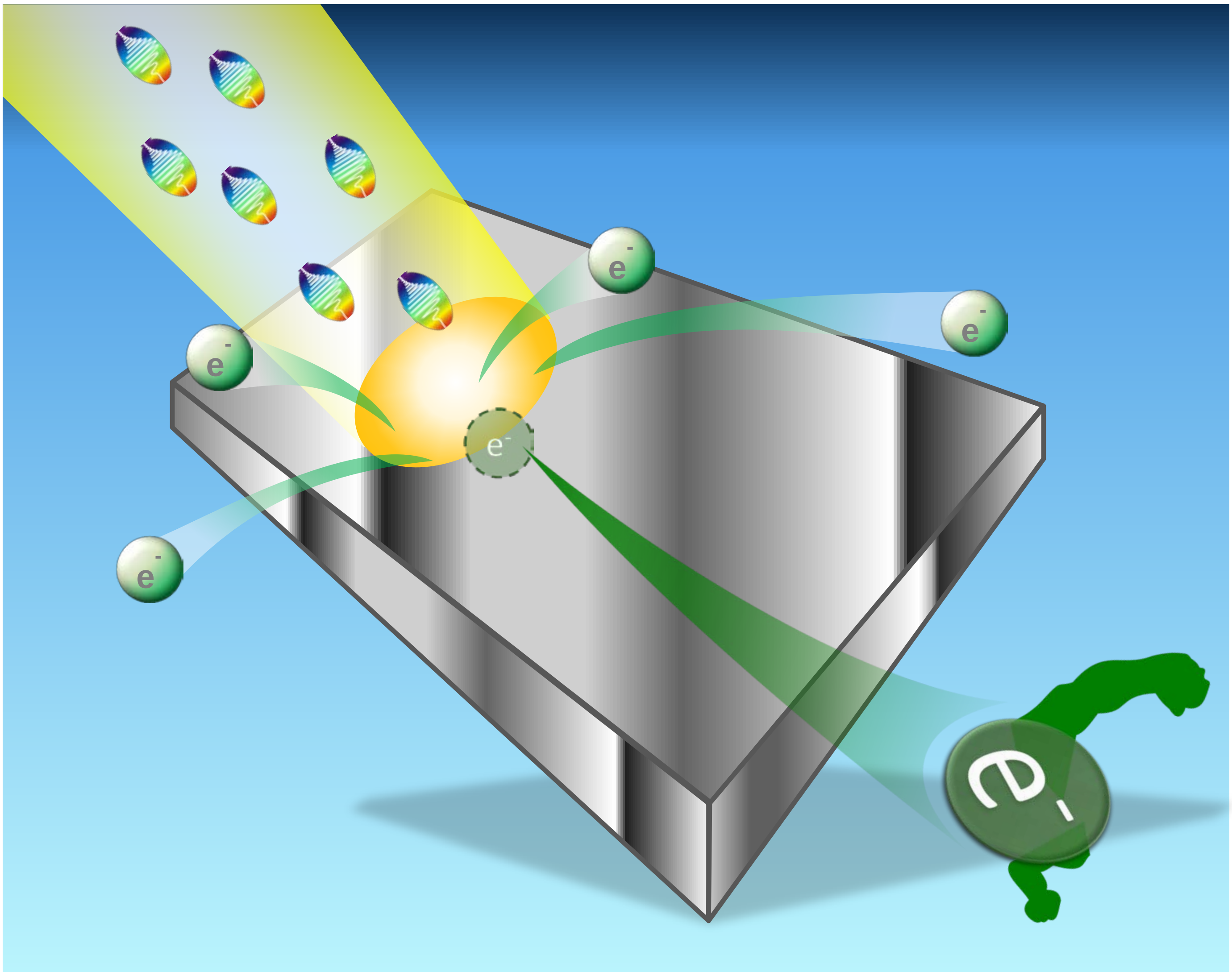
H2 Topic 18a



Quantum Physics (I):

The Photoelectric Effect

Content

- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle
- Schrodinger model
- Barrier tunnelling

Learning Outcomes

Candidates should be able to:

(a) show an appreciation of the particulate nature of electromagnetic radiation.

(b) recall and use $E = hf$.

(c) show an understanding that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature.

(d) recall the significance of threshold frequency.

(e) recall and use the equation $\frac{1}{2}mv_{max}^2 = eV_s$, where V_s is the stopping potential.

(f) explain photoelectric phenomena in terms of photon energy and work function energy.

(g) explain why the maximum photoelectric energy is independent of intensity whereas the photoelectric current is proportional to intensity.

(h) recall, use and explain the significance of $hf = \Phi + \frac{1}{2}mv_{max}^2$.

(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles.

(j) recall and use the relationship for the de Broglie wavelength $\lambda = h/p$.

(k) show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to spectral lines.

(l) distinguish between emission and absorption line spectra.

(m) recall and solve problems using the relation $hf = E_1 - E_2$.

(n) explain the origins of the features of a typical X-ray spectrum using quantum theory.

(o) show an understanding of and apply the Heisenberg position-momentum and time-energy uncertainty principles in new situations or to solve related problems.

(p) show an understanding that an electron can be described by a wave function Ψ where the square of the amplitude of wave function $|\Psi|^2$ gives the probability of finding the electron at a point. (No mathematical treatment is required.)

(q) show an understand of the concept of a potential barrier and explain qualitatively the phenomenon of quantum tunnelling of an electron across such a barrier.

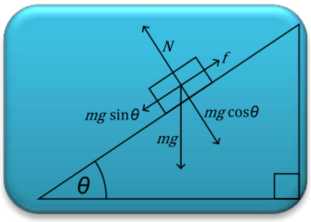
(r) describe the application of quantum tunnelling to the probing tip of a scanning tunnelling microscopy (STM) and how this is used to obtain atomic-scale images of surfaces. (Details of the structure and operation of a scanning tunnelling microscope are not required.)

(s) apply the relationship transmission coefficient $T \propto exp(-2kd)$ for the STM in related situations or to solve problems. (Recall of the equation is not required.)

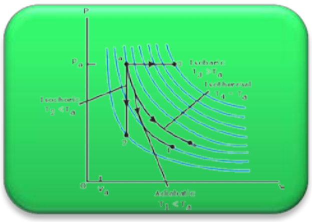
(t) recall and use the relationship $R + T = 1$, where R is the reflection coefficient and T is the transmission coefficient, in related situations or to solve problems.

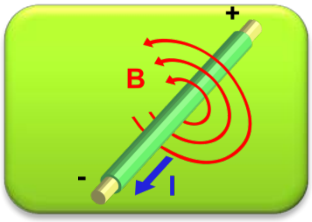
18a.0 Introduction

By the end of the 19th century, there was a certain smugness amongst Physicists. Physics allowed us to predict trajectories of projectiles and celestial bodies, build engines to aid work, and permitted wireless telecommunications to flourish. Everything could have been deterministically certain; or so they thought.

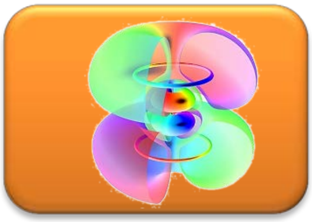


Mechanics


Thermodynamics



Electromagnetism


(??)

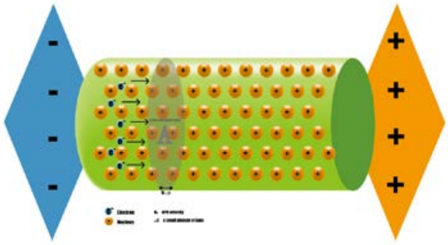
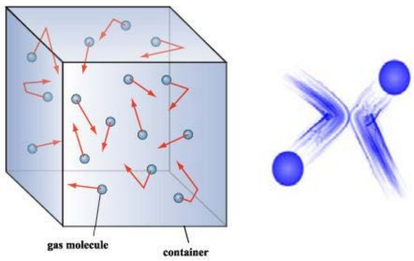
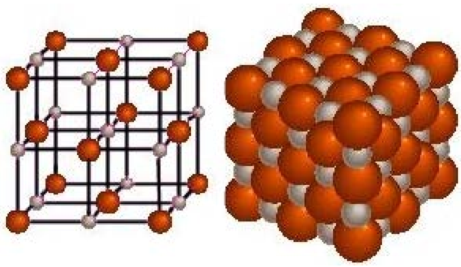
However, there existed some experimental results which could not be explained under the then-available theories. Physicists involved in such projects were perplexed to no end.

Here, we visit a pivotal experiment, carried out by Hungarian physicist Philipp Lenard in 1902, which had results that defied the conventional wisdom of those days. And in so doing, we trace the roots of Quantum Physics.

18a.1 The 2 Models

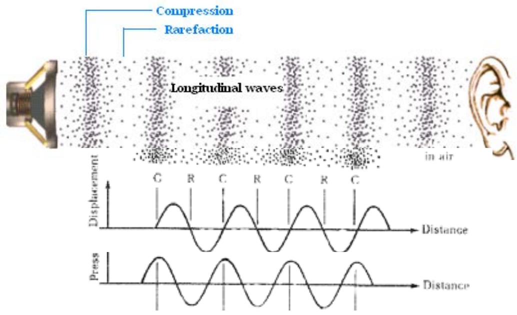
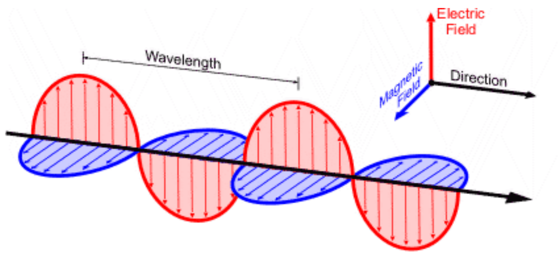
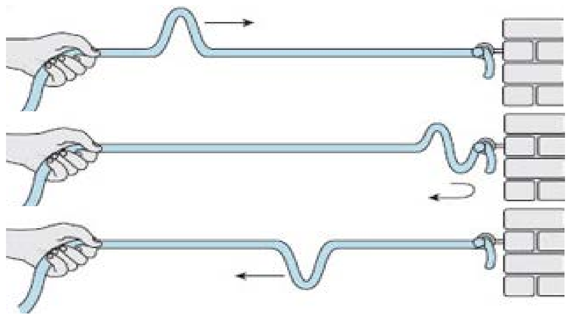
Here, we review 2 scientific models which have been serving our purposes separately up to this point in our Physics journey: the ***particle*** model and the ***wave*** model.

When describing matter, we often think about ***particles*** as small masses that are hard, rigid and move about according to Newton's Laws of motion.

area	microscopic model	macroscopic phenomena	typical diagrams
electricity	flow of electrons	current	
gases	Kinetic Theory	pressure, volume, and temperature of gas	
solids	lattice structure	mechanical properties	

We often can explain the macroscopic phenomena by the random motions and collisions of the particles. The collisions between particles, are also often said to conserve linear ***momentum***.

On the other hand, waves are used to explain the transport of energy without matter being transported. The medium of transport is often described to be ***oscillating***.

area	oscillating quantity	typical diagrams
sound	air pressure	
light *or* electromagnetic radiation	electric field *and* magnetic field	
waves on string	displacement (of string)	

As our studies in the previous of topics of superposition has shown, light can undergo ***superposition*** and ***interference***, which are strong evidences of light having a wave nature.

18a.2 Light Can Behave As Particles

(a) *show an appreciation of the particulate nature of electromagnetic radiation.*

(b) *recall and use E = hf.*

Instead of treating electromagnetic radiation as a continuous wave, both Max Planck and Albert Einstein independently postulated EM radiation as discrete packets of energy which behave like particles.

The Photon
A quantum of energy that is electromagnetic radiation.
The energy E of a single photon is given by $$E = hf$$ where f is the frequency of the electromagnetic radiation h is Planck's constant (= 6.63×10^{-34} Js)

There are several observations we can make

1. $E = hf$

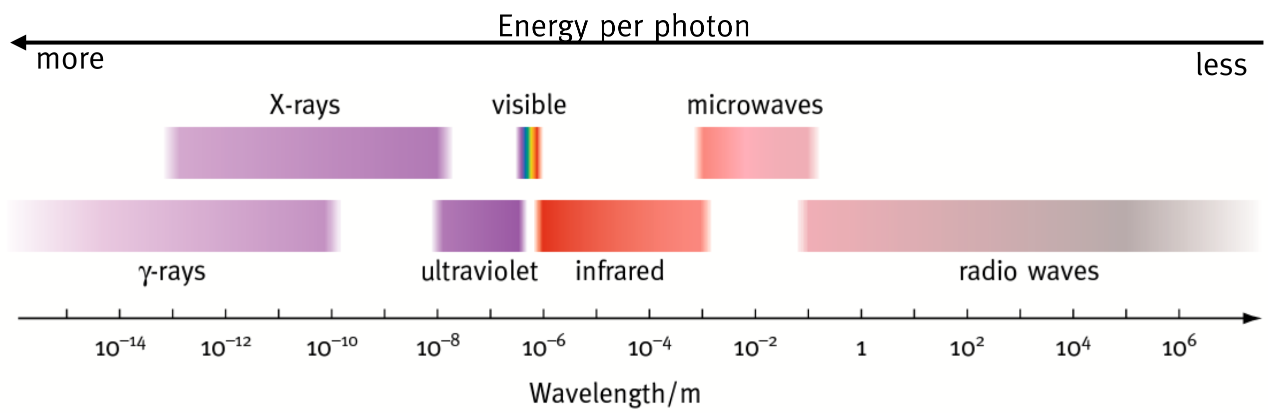
   The equation tells us the relationship between a *particle* property (the photon energy) and a wave property (the frequency).

2. $E \propto f$

   The energy of a single photon is directly proportional to the frequency. The higher the frequency of the radiation, the higher the energy of each photon.

3. $E = \dfrac{hc}{\lambda}$

   By invoking $c = f\lambda$, the energy of a photon is inversely proportional to the wavelength. Hence, a short-wavelength X-ray photon contains more energy than the long-wavelength microwave photon.



Example 1

Determine the energy of a high-energy γ-photon of frequency 10^{26} Hz.

Solution:

$$E = hf$$
$$= (6.63 \times 10^{-34})(10^{26})$$
$$= 6.63 \times 10^{-8} \text{ J}$$

Example 1 shows that the photon energy even for a "high-energy" photon, is far less than 1 J. Hence, the joule is not a convenient unit for measuring photon energies. The **electronvolt (eV)** is often used for such purposes.

1 eV is the energy transferred when an electron travels through a potential difference of 1 volt:

$$W = Q\Delta V$$
$$= (1.6 \times 10^{-19})(1)$$
$$= 1.6 \times 10^{-19} \text{ J}$$

Therefore, 1 eV =1.6×10^{-19} J.

- To convert from eV to J, multiply by 1.6×10^{-19}
- To convert from J to eV, divide by 1.6×10^{-19}

Example 2

Visible light has wavelengths spanning from 400 nm (violet) to 700 nm (red). Find the energy, in eV, of (i) a photon of red light and (ii) a photon of violet light.

Solution:

(i) For red light:

$$E_{red} = hf$$
$$= \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{700 \times 10^{-9}}$$
$$= 2.84 \times 10^{-19} \text{ J}$$
$$= 1.78 \text{ eV}$$

(ii) For violet light:

$$E_{violet} = hf$$
$$= \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{400 \times 10^{-9}}$$
$$= 4.97 \times 10^{-19} \text{ J}$$
$$= 3.11 \text{ eV}$$

*NB: Ultra**violet** causes skin cancer while infra**red** causes heating.*

Per photon, "blue-er" photons are more energetic than "red-der" photons.

Example 3

Determine the conversion factor for converting wavelength (nm) into energy (eV).

Solution:

Energy of 1 photon:

$$E = hf = \frac{hc}{\lambda}$$

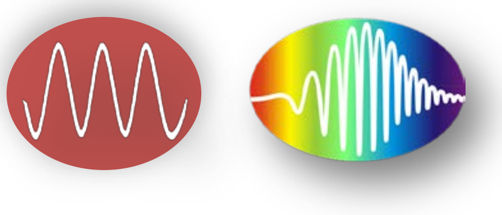
For conversion into eV :

$$E_{\text{eV}} = \frac{hc}{\lambda e}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{\lambda_{\text{nm}}(10^{-9})(1.6 \times 10^{-19})}$$

$$= \frac{1243}{\lambda_{\text{nm}}}$$

Graphical depictions of the particulate nature of EM radiation typically try to encapsulate the wave-nature into a “packet” of energy:



Some commonly-seen graphical representations depicting a photon

Example 4

A 1.0 mW laser produces red light of wavelength 663 nm. Calculate how many photons the laser produces per second.

Solution:

$$E_{photon} = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{663 \times 10^{-9}}$$

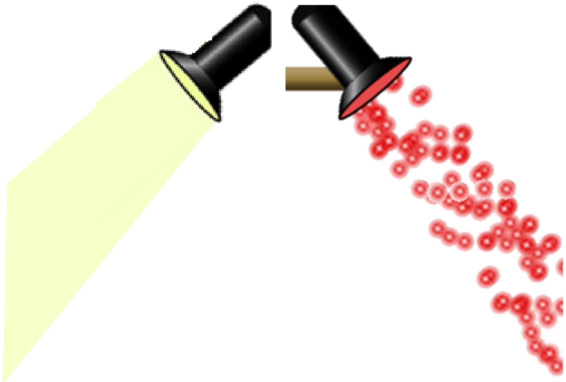
$$= 3 \times 10^{-19}\ \text{J}$$

$$E_{total} = Pt = nE_{photon}$$

$$\frac{n}{t} = \frac{P}{E_{photon}}$$

$$= \frac{1.0 \times 10^{-3}}{3 \times 10^{-19}}$$

$$= 3.33 \times 10^{15}\ \text{s}^{-1}$$



dual nature of light

Example 4 Extension

The same 1.0 mW laser is then tuned to produce UV light of wavelength 221 nm instead. Calculate how many UV photons the laser now produces per second.

Solution:

$$E_{photon} = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{221 \times 10^{-9}}$$

$$= 9 \times 10^{-19}\ \text{J}$$

$$E_{total} = Pt = nE_{photon}$$

$$\frac{n}{t} = \frac{P}{E_{photon}}$$

$$= \frac{1.0 \times 10^{-3}}{9 \times 10^{-19}}$$

$$= 1.11 \times 10^{15}\ \text{s}^{-1}$$

NB: For the same power output, there can be more photons-per-second of lower-energy, or less photons-per-second of higher-energy.

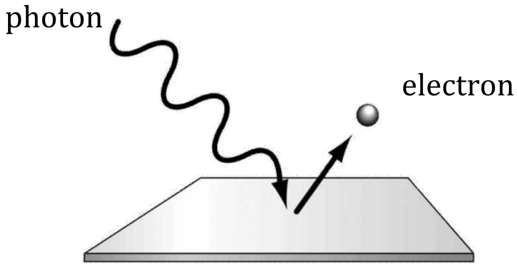
18a.3 Einstein's Photoelectric Equation

(d) *recall the significance of threshold frequency.*

(f) *explain photoelectric phenomena in terms of photon energy and work function energy.*

(h) *recall, use and explain the significance of $hf = \Phi + ½mv^2_{max}$.*

It was found that when light of sufficiently high frequency is incident on a metal surface, electrons can be emitted. This is known as the *photoelectric effect* and we call such emitted electrons *photoelectrons*. In 1905, Albert Einstein came up with an explanation for this phenomenon based on the idea of photons.



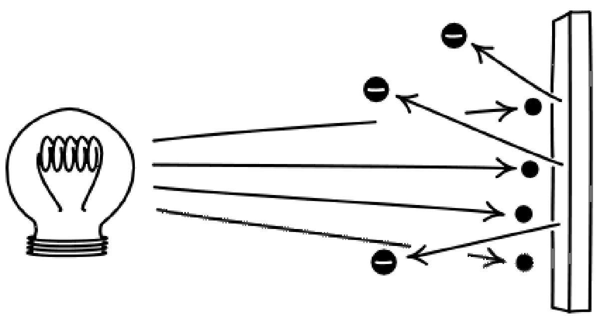
The Photoelectric Equation

$$hf \quad = \quad \Phi \quad + \quad KE_{max}$$

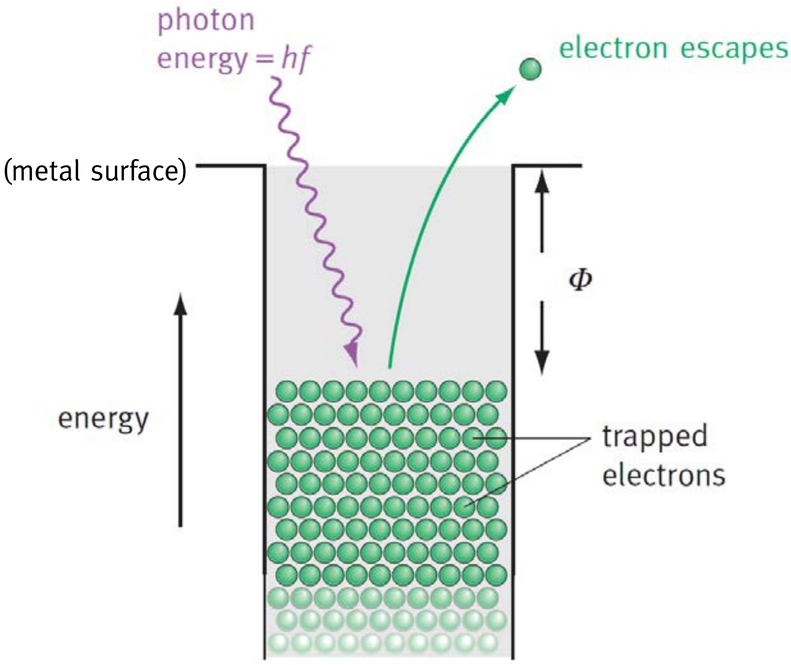
Energy provided by a single photon	=	***Work function*** of a metal	+	***Maximum kinetic energy*** of liberated photoelectron

Some characteristics of the photoemission:

- A single photon can only interact (and pass on its energy) to a single electron.
  It is a one-to-one interaction
- Not all photons (of sufficient energy) get to interact with electrons.
  There is probability involved for a successful interaction.
- The photoelectrons are emitted in all random directions with varying speeds.



The photoelectric equation is a statement which echoes the principle of conservation of energy.



Energy is needed to release the electrons because they are attracted by electrostatic forces due to the lattice of positive metal ions.

Example 5

Light of 543 nm is incident on a clean sodium surface. The photoelectrons released are found to have negligible amounts of kinetic energy. Determine the work function Φ for sodium metal in eV.

Solution:

$$hf = \Phi + KE_{max}$$

$$\Phi_{Na} = \frac{hc}{\lambda} - KE_{max}$$

$$\Phi_{Na} = \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{543 \times 10^{-9}} - 0$$

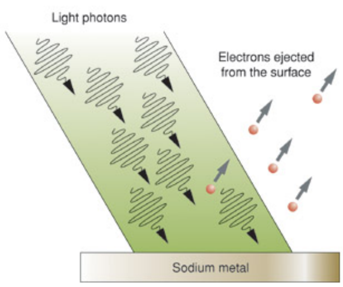
$$= 3.65 \times 10^{-19}\ \text{J}$$

$$= 2.28\ \text{eV}$$

Work Function Φ

The minimum energy required to remove an electron from a metal surface.

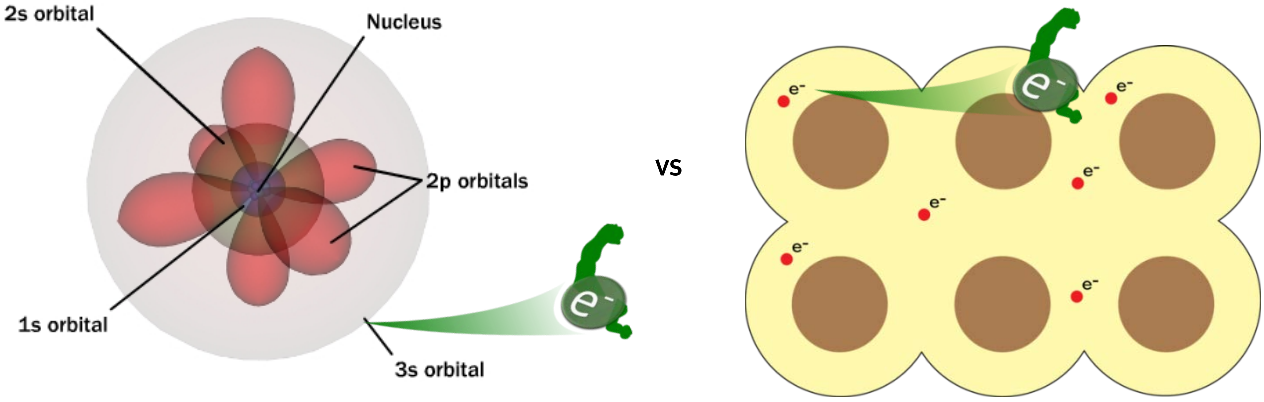
Φ is a property of the metal and is unique to each type of metal. It is typically measured in eV.



For the Chemistry students…

The idea behind *ionization energy* and *work function* might seem similar since both involve the removal of electron(s) from the system. However, *ionization* involves gaseous, isolated atoms in ground state; while *photoemission* is from a sea of delocalised mobile electrons which permeates a regular lattice of metal cations.

Due to the unique properties of metallic bonding structure, the work function is less energetically demanding as compared to the ionization energy for the same species of metal.



Example 6

Magnesium metal has a work function of 3.63 eV. Find the minimum frequency of electromagnetic radiation required for electron to be emitted from the surface. Determine the corresponding wavelength.

Solution:

$$hf = \Phi + KE_{max}$$

For minimum frequency:

$$\Phi_{Mg} = hf - 0$$

$$f = \frac{\Phi_{Mg}}{h}$$

$$= \frac{(3.63)(1.6 \times 10^{-19})}{6.63 \times 10^{-34}}$$

$$= 8.76 \times 10^{14} \text{ Hz}$$

The corresponding wavelength:

$$c = f\lambda$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^{8}}{8.76 \times 10^{14}}$$

$$= 342 \times 10^{-9} \text{ m}$$

*NB: due to the inverse relationship of $f \propto \frac{1}{\lambda}$, this is the **maximum** wavelength that can cause photoemission*

Threshold Frequency f_0

minimum frequency of electromagnetic radiation
for electron to be emitted from metal surface

Threshold Wavelength λ_0

maximum wavelength of electromagnetic radiation
for electron to be emitted from metal surface

$\Phi = hf_0$	$\Phi = \dfrac{hc}{\lambda_0}$

Example 7

The maximum kinetic energy of the electrons emitted from a metallic surface is 1 eV when the frequency of the incident radiation is 7.5×10^{14} Hz. Determine

(i) the work function of the metal in eV;

(ii) the threshold wavelength of this metal.

Solution:

(i)

$$hf = \Phi + KE_{max}$$

$$\Phi = hf - KE_{max}$$

$$= (6.63 \times 10^{-34})(7.5 \times 10^{14}) - (1)(1.6 \times 10^{-19})$$

$$= 3.37 \times 10^{-19} \text{ J}$$

$$= 2.11 \text{ eV}$$

(ii)

$$\Phi = hf_0 = \frac{hc}{\lambda_0}$$

$$\lambda_0 = \frac{hc}{\Phi}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(2.11)(1.6 \times 10^{-19})}$$

$$= 589 \text{ nm}$$

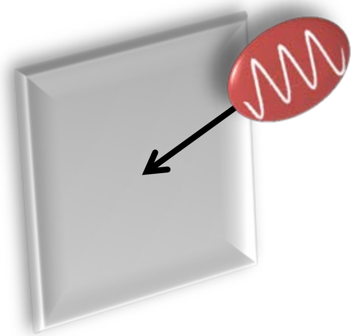
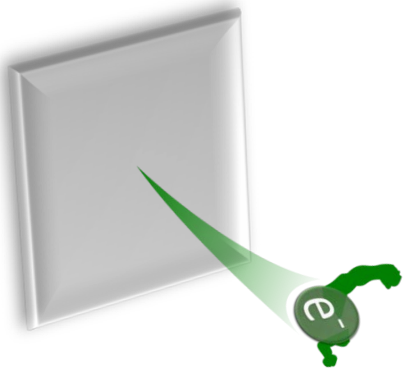
18a.7 The Kinetic Energy of the Photoelectron

(e) *recall and use the equation* $\frac{1}{2}mv^2_{max} = eV_s$, *where* V_s *is the stopping potential.*

(g) *explain why the maximum photoelectric energy is independent of intensity whereas the photoelectric current is proportional to intensity*

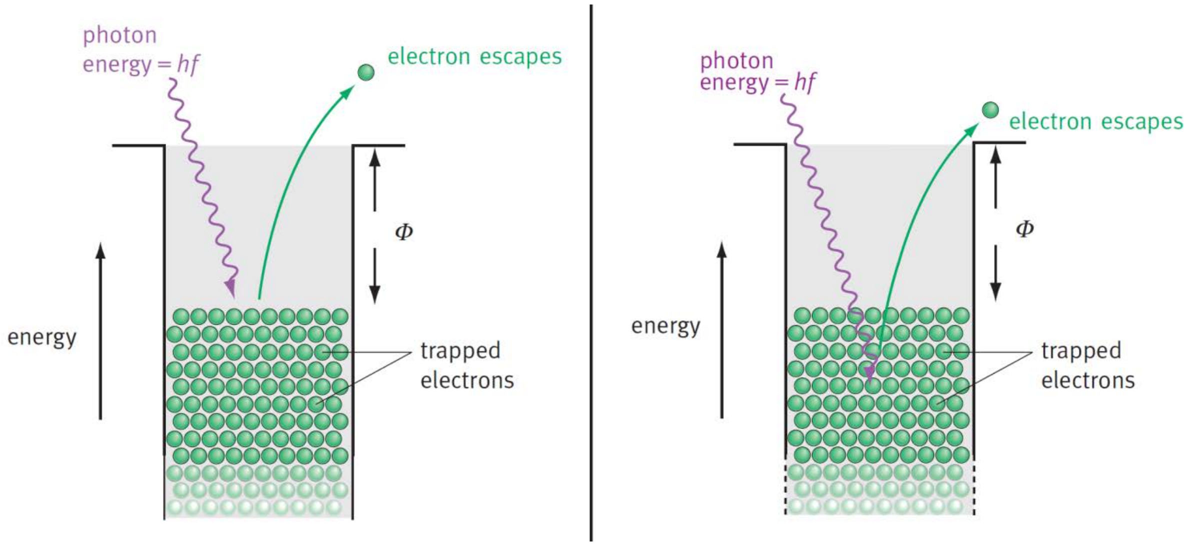
(h) *recall, use and explain the significance of* $hf = \Phi + \frac{1}{2}mv^2_{max}$.

The energy conservation can be understood in the following manner:

 A photon of energy hf is incident on the metal surface.	 Photon meets an electron. Electron absorbs the energy.	 Some of the energy is used to escape from the metal. The rest remains as KE of the electron.

The photoelectric equation $hf = \Phi + KE_{max}$ describes situations where electrons nearest to the metal surface are removed. Since they are the least-bound to the metal, they will possess the most amounts of kinetic energy for the same amount of photon energy.

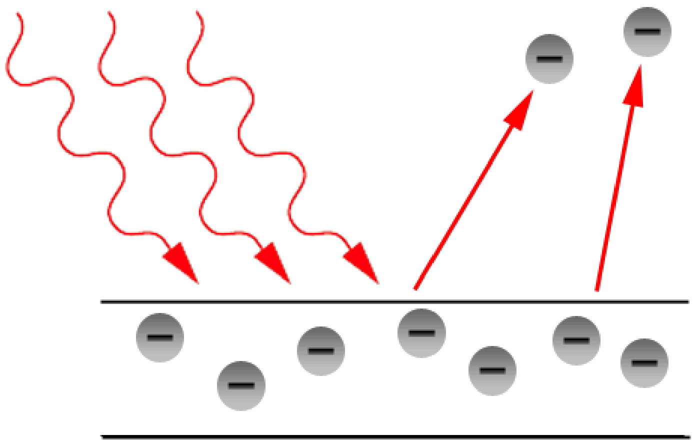
Because electrons of different "depths" can absorb incoming photons, the emission results in photoelectrons with a ***range of kinetic energies***.



If the photon energy is not enough to remove an electron from the metal, it can either (i) pass through the metal or (ii) be absorbed as kinetic energy of the electrons which collide randomly with the lattice of positive metal ions, increasing the temperature.

Why was it necessary to involve the particle model instead of the wave model to explain the photoelectric effect? The table below summaries the experimental observations, the predictions from wave model and how the photon model was about to account for the observations better.

Observation	Wave model prediction	Photon model explanation
Instantaneous emission of electrons as soon as light shines on metal	Very intense (high power) light should be needed to have immediate effect	A single photon is enough to release one electron
Low intensity light can cause electrons to be emitted	There should be a time lag for the metal surface to accumulate enough energy	Low-intensity light means fewer photons per second. Emission of electrons can still occur, just less frequently.
Increasing light intensity increases the rate at which electrons leave the metal	Higher light intensity should mean electrons having more energy	Greater intensity means more photons, not photons having more energy each.
A minimum threshold frequency of light is needed	Any frequency will causes emission of electron if exposure time is long enough	A photon from low-frequency light has energy too small to release an electron
Increasing light-frequency results in electrons having more KE_{max}	It should be increasing intensity that increases the energy of electrons, not frequency	A photon from high-frequency light has more energy, so emitted electrons can gain more KE



Example 8

Photons of energies 1.0 eV, 2.0 eV, 3.0 eV strike a metal surface of work function 1.8 eV.

(i) State which photons will be able to cause emission of electrons from the metal surface.

(ii) Determine the maximum kinetic energies, in joules, of the electrons emitted (if any).

Solution:

$$hf = \Phi + KE_{max}$$

For photoemission to occur, $E_{photon} > \Phi$

Photons of 2 eV and 3 eV will causes electrons to be emitted.

For the 2 eV photon:

$$KE_{max} = E_{photon} - \Phi$$
$$= 2 - 1.8 = 0.2\ eV$$
$$= 3.2 \times 10^{-20}\ \text{J}$$

For the 3 eV photon:

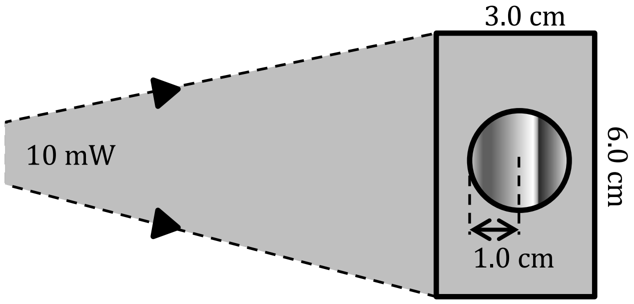
$$KE_{max} = 1.2\ \text{eV} = 1.92 \times 10^{-19}\ \text{J}$$

Example 9

10 mW of light of wavelength 532 nm is uniformly-incident on a rectangular area measuring 3.0 cm by 6.0 cm.

(i) Determine the intensity of the light across the above-mentioned rectangular area.

(ii) A flat, circular piece of metal with a radius of 1.0 cm is placed in the illuminated region. Find the power of the light that is incident on the metal.



Solution:

(i) $I = \frac{P}{A}$

$I_{lit} = \frac{P_{output}}{A_{lit}} = \frac{10\times10^{-3}}{(3\times10^{-2})(6\times10^{-2})} = 5.6\ \text{Wm}^{-2}$

(ii) ***Method 1***

$$P_{captured} = I_{lit}A_{captured} = I_{lit}(\pi r^2)$$
$$= (5.6)(\pi)(1 \times 10^{-2})^2 = 1.8\ \text{mW}$$

Method 2

By ratio

$$\frac{P_{captured}}{P_{total}} = \frac{A_{captured}}{A_{lit}}$$

$$P_{captured} = P_{total}\frac{A_{captured}}{A_{lit}}$$

$$= 10\frac{\pi(1 \times 10^{-2})^2}{(3 \times 10^{-2})(6 \times 10^{-2})} = 1.7\ \text{mW}$$

*(*round-off error in between the 2 values)*

Example 10

Violet light of 400 nm is incident on the surface of a potassium metal, work function 2.0 eV. Determine the maximum speed of electrons emitted from the surface.

Solution:

$$hf = \Phi + KE_{max}$$

$$\frac{hc}{\lambda} = \Phi + \frac{1}{2} m_e v_{max}^2$$

$$v_{max} = \sqrt{\frac{2\left(\frac{hc}{\lambda} - \Phi\right)}{m_e}}$$

$$= \sqrt{\frac{2\left(\frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{400 \times 10^{-9}} - (2)(1.6 \times 10^{-19})\right)}{9.11 \times 10^{-31}}} = 6.23 \times 10^{5} \text{ ms}^{-1}$$

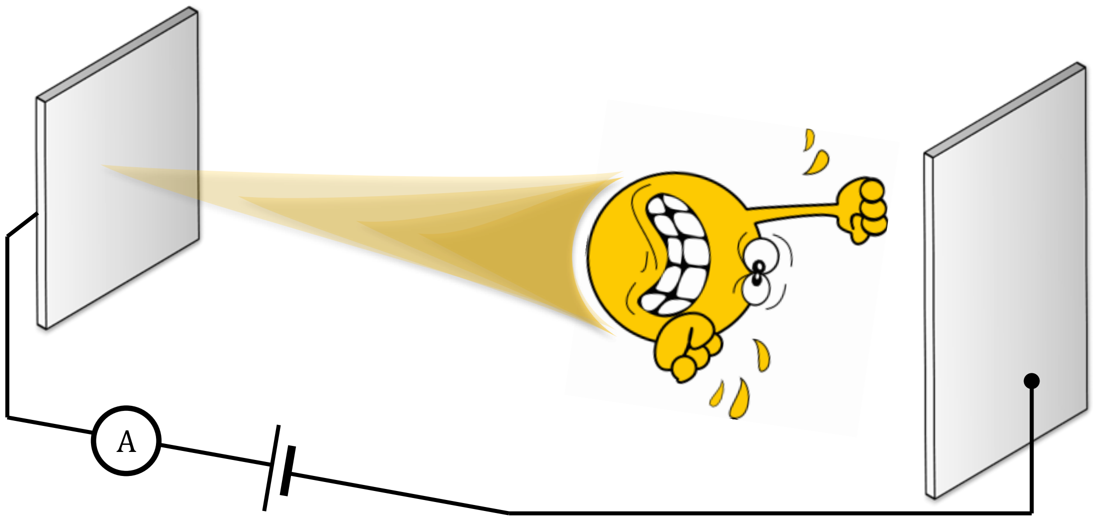
NB: be sure to check that such answers for velocity do not exceed the speed of light

Since electrons are invisible to the naked eye, and the velocities involved are large, there needs to be a convenient way of measuring the kinetic energies of the emitted electrons.

Stopping Potential V_s

$$\frac{1}{2} m v^2 = eV_s$$

The negative potential which prevents the most energetic photoelectrons from reaching the collector plate, resulting in zero photoelectric current.



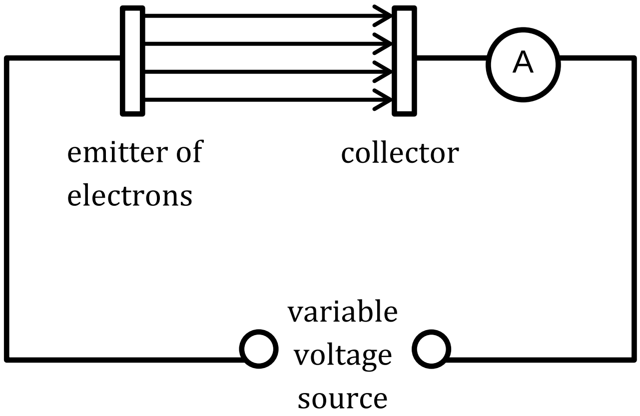
Example 11

In a particular experiment, it was found that a potential difference of 2.10 V was sufficient to register no current in the ammeter. Electrons are ejected from the emitter with varying amounts of kinetic energy.

(i) Draw the electric field lines by treating the emitter and collector as a pair of parallel plates.

(ii) Find the maximum kinetic energy with which the electrons are emitted.

(iii) Hence, determine the speed of the fastest electrons.



Solution:

(ii)

By Principle of Conservation of Energy,

kinetic energy = work done against electric field

$$W = Q\Delta V$$

$$KE_{max} = eV_s$$

$$=(1.6\times10^{-19})(2.10)$$

$$= 3.36 \times 10^{-19} \text{ J}$$

(iii)

$$\frac{1}{2}mv^2 = eV_s$$

$$v_{max} = \sqrt{\frac{2eV_s}{m_e}}$$

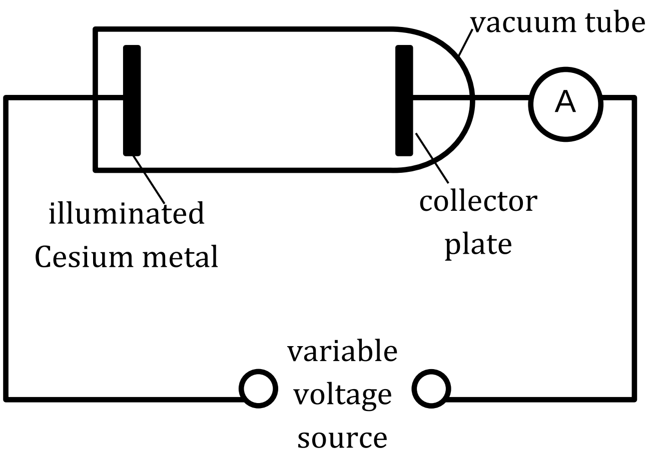
$$= \sqrt{\frac{2(1.6 \times 10^{-19})(2.1)}{9.11 \times 10^{-31}}}$$

$$= 859 \times 10^5 \text{ms}^{-1}$$

NB: If the space between the metal plates isn't vacuum, the emitted electrons will undergo collisions with other particles.

Example 12

UV light of wavelength 350 nm falls onto Cesium metal of work function 2.1 eV. Find the stopping potential required for the ammeter to register zero current on the ammeter.



Solution:

Photoelectric equation:

$$hf = \Phi + KE_{max}$$

maximum kinetic energy in eV:

$$KE_{max} = \frac{hc}{\lambda e} - \Phi_{Cs}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(350 \times 10^{-9})(1.6 \times 10^{-19})} - (2.1)$$

$$= 3.55 - 2.1$$

$$= 1.45 \text{ eV}$$

Stopping potential:

$$KE_{max} = eV_s$$

$$1.45(e) = eV_s$$

$$V_s = 1.45 \text{ V}$$

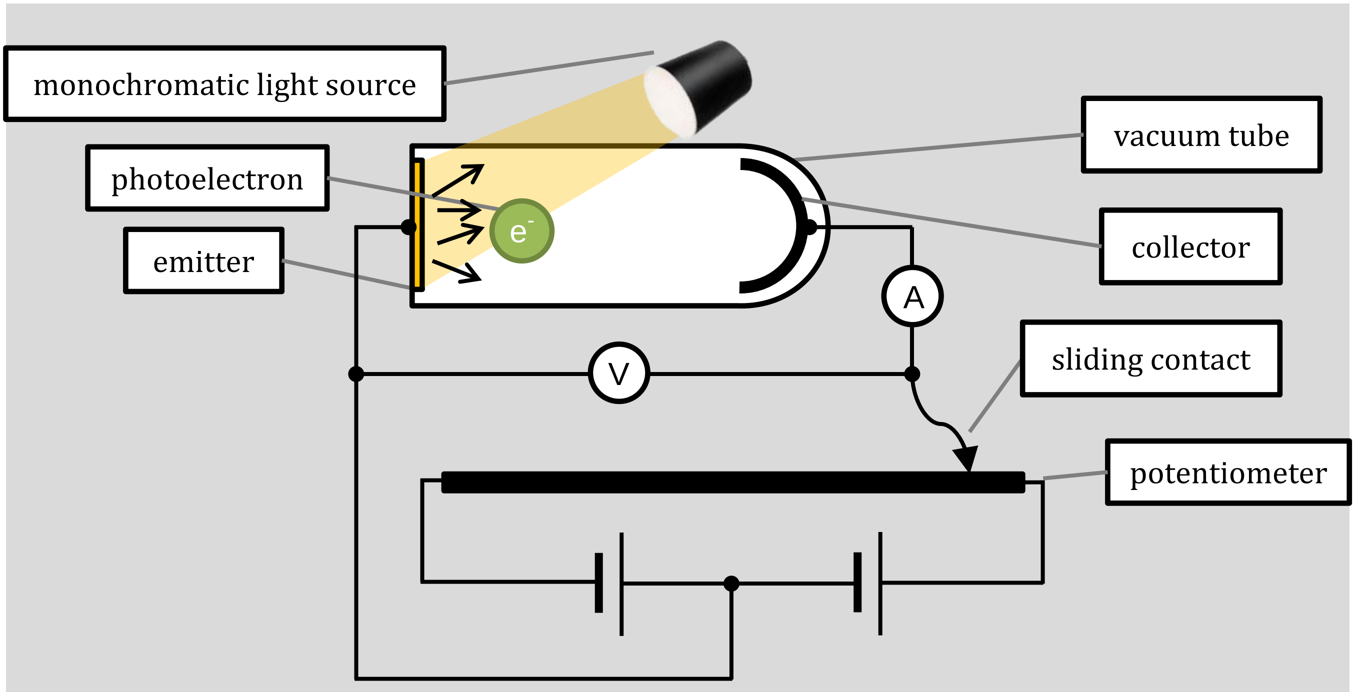
NB: The stopping potential is a ***negative*** *voltage; it needs to "repel" the emitted electron.*

18a.8 The Experimental Set-up

(g) explain why the maximum photoelectric energy is independent of intensity whereas the photoelectric current is proportional to intensity

The figure below shows a typical photoelectric experiment set-up. As developed from Examples 9 and 10, the metal from which photoelectric effect is observed is generically called as the "emitter". It is housed in a transparent vacuum tube together with the "collector".

A potential divider (c.f. topic on DC circuits) typically serves as the variable voltage source.



Apparatus	Function
Monochromatic light source	Provides the electromagnetic radiation that shines on the emitter plate. The output is of well-defined wavelength (*mono-chromat*ic means "*single colour*").
Vacuum tube	Sometimes otherwise called an *evacuated tube*, the vacuum inside the tube allows movement of electrons to be free of collisions with gas molecules.
Emitter	Electrons are ejected from this metallic surface when electromagnetic radiation of sufficient photon energy falls on it.
Collector	A variable potential difference is supplied across the collector and the emitter. Electrons emitted from the emitter plate can be made to move towards or move away from the collector plate by varying the potentiometer.
Photoelectron	Electrons which are ejected from the surface of the metallic emitter after absorbing sufficient energy in the form of electromagnetic radiation. *Reminder: Photoelectrons are emitted in random directions with various speeds.*

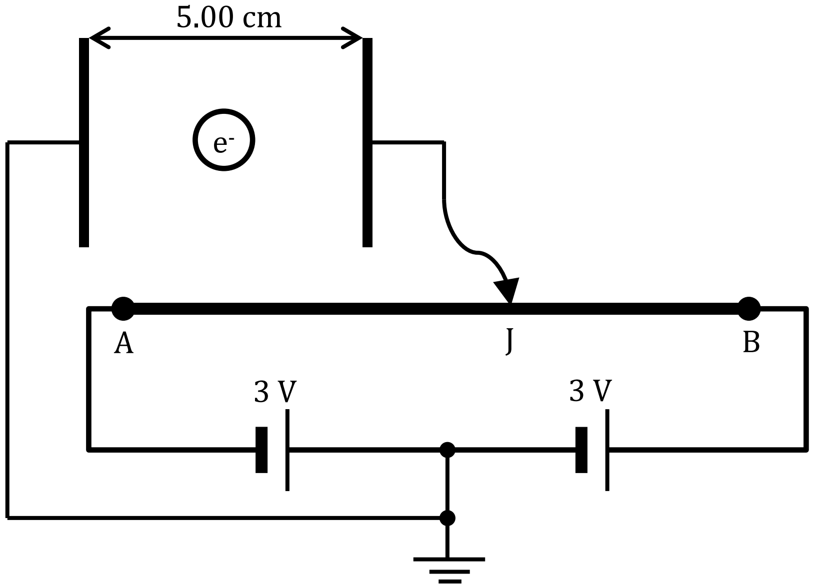
Example 13

Resistance-wire AB is 1.00 m in length, and is connected to two 3V cells in the circuit below. Find the force acting on an initially-stationary electron between the metal plates when length AJ is

(i) 0.20 m

(ii) 0.50 m

(iii) 0.75 m



Solution:

The left-hand plate is always at 0V since it is earthed. By potential divider rule,

$$\frac{L_{AJ}}{L_{AB}} = \frac{V_{AJ}}{V_{AB}} \quad \rightarrow \quad V_{AJ} = V_{AB}\left(\frac{L_{AJ}}{L_{AB}}\right)$$

$$= [3 - (-3)]\left(\frac{L_{AJ}}{1.00}\right)$$

$$V_{AJ} = 6L_{AJ}$$

Since $V_A = -3$ V, the potential at J, V_J is:

$$V_J = V_{AJ} - 3$$

$$= 6L_{AJ} - 3$$

And the electric force acting on the electron is given by:

$$F_e = QE$$

$$= e\left(\frac{\Delta V}{d}\right)$$

Thus

$$|F_e| = \left|\left(\frac{1.6 \times 10^{-19}}{5 \times 10^{-2}}\right)[V_j - 0]\right|$$

$$= \left|(9.6 \times 10^{-18})(2L_{AJ} - 1)\right|$$

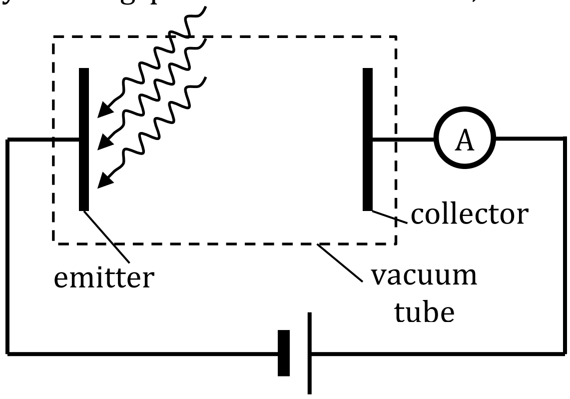
(i) For $L_{AJ} = 0.20$ m,	(ii) For $L_{AJ} = 0.50$ m,	(iii) For $L_{AJ} = 0.75$ m,
$V_j = -1.80$ V	$V_j = 0$ V	$V_j = +1.5$ V
$\lvert F_e \rvert = 5.76 \times 10^{-18}$ N	$\lvert F_e \rvert = 0$ N	$\lvert F_e \rvert = 4.8 \times 10^{-18}$ N
Force on electron acts to left.	No electric force on electron.	Force on electron acts to right.

Example 14

Light of wavelength 300 nm falls onto a metal emitter. The photon energy is enough to cause emission of photoelectrons. The ammeter reads 0.08 mA.

(i) Calculate the number of electrons being emitted per second.

(ii) If the probability of a photon successfully causing photoemission is 2.3%, estimate the power of the light shining on emitter.



Solution:

There can be current flowing throughout only when photoelectric effect is active;
and since the collector plate is positively-biased with respect to the emitter,
we assume that all the emitted electrons are being attracted to the collector plate.

(i)

$$I = \frac{Q}{t} = \frac{ne}{t}$$

The number of electrons emitted per second:

$$\frac{n}{t} = \frac{I}{e} = \frac{0.08 \times 10^{-3}}{1.6 \times 10^{-19}} = 5.0 \times 10^{14}\ \text{s}^{-1}$$

(ii)

The number of photons reaching the plate per second,

$$\frac{n_{photon}}{t} = \frac{n_{electron}}{2.3\%}$$

Photon energy,

$$E = hf = \frac{hc}{\lambda}$$

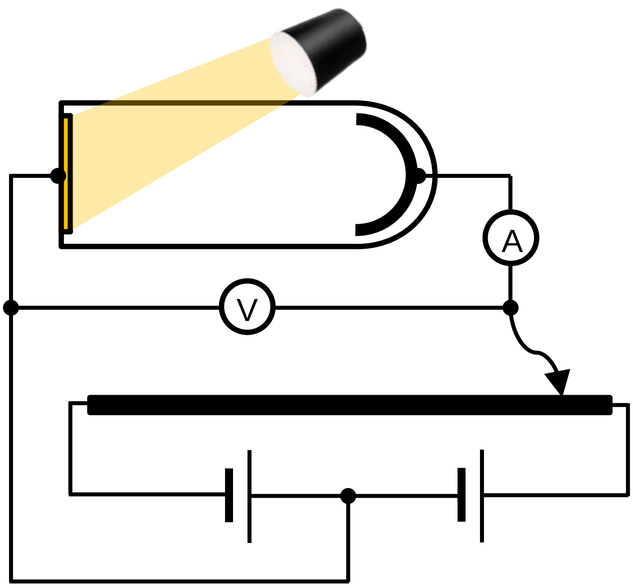
Power,

$$P = \frac{E_{total}}{t} = \frac{n_{photon}}{t}E = \frac{n_{photon}}{t}\left(\frac{hc}{\lambda}\right) = \frac{5.0 \times 10^{14}}{2.3\%}\left(\frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{300 \times 10^{-9}}\right) = 14.4 \times 10^{-3}\ \text{W}$$

NB: The "photoelectric current" reflects the charge-per-sec of the electrons reaching collector. If all of the electrons are captured (when the collector is positive w.r.t. the emitter), then the photocurrent is directly proportional to the number of photons shining on the emitter.

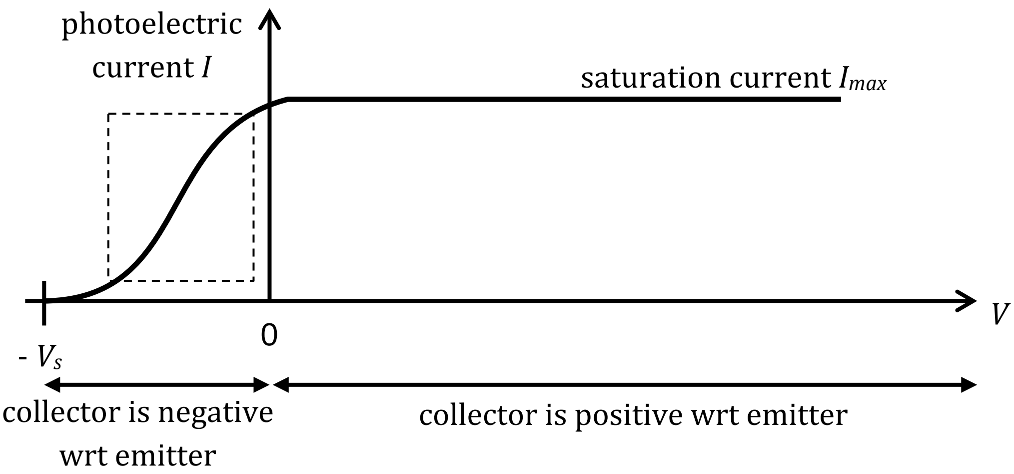
Question 1: What should I observe if I change the p.d. between the emitter and collector plate?

The potential difference between the plates is changed by sliding the jockey in the circuit here:



At the stopping potential V_s, even the electrons with the most kinetic energy cannot reach the negatively-biased collector.

When the collector is positively-biased, all emitted electrons are captured. The "saturation current" is limited by the number of incoming photons.

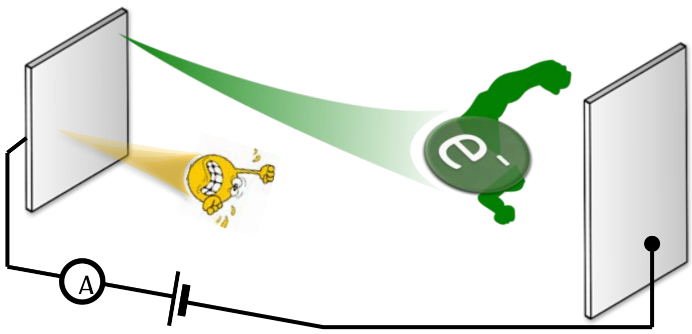


Question 1a: What is the significance of the region bounded by the dotted box?

The photoelectric current is between zero and the saturation current.

Only some electrons are reaching the collector plate.

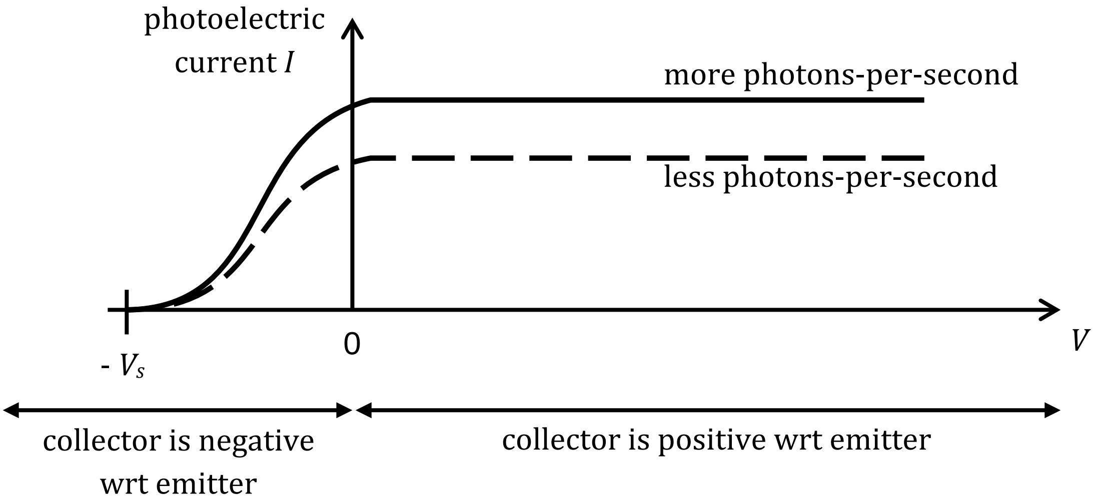
Electrons are emitted with a range of kinetic energies; only those with enough can do work against the electric field to reach the collector.



Question 1b: What if I increase the number of same-frequency photons reaching the emitter per second?

This is a more-powerful light source operating at the same colour. More photons reaching per second results in more emitted electrons per second. The graph in Q1 will have a higher saturation current.

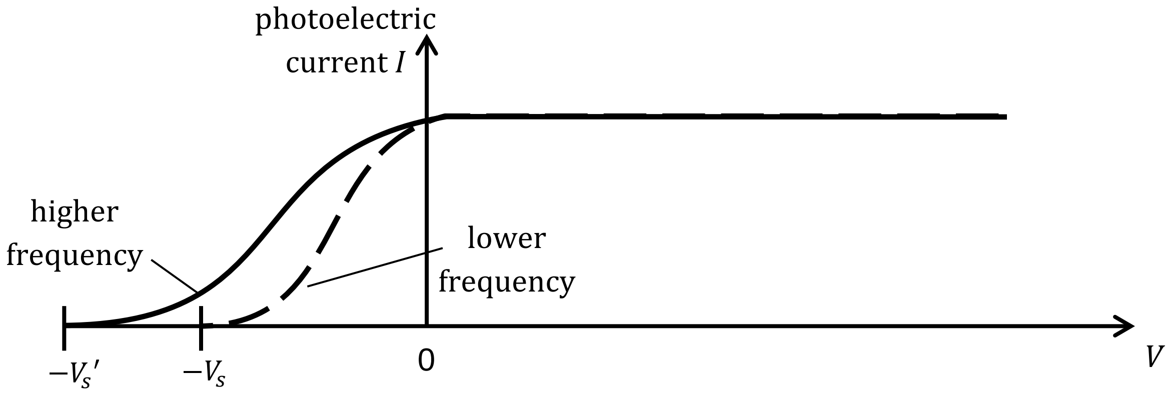
However, since the photon energy remains the same, the maximum KE, and conversely the stopping potential V_s remains the same.



Question 1c: What if I increase the light-frequency but keep the number of photons reaching the plate per second?

The photon energy, given by $E = hf$, will be more. From the photoelectric equation $hf = \Phi + KE_{max}$, the maximum kinetic energy of the emitted electrons will be higher. A larger stopping potential V_s will be required to prevent these electrons from reaching the collector plate.

However, since the number of photons-per-second remains the same, number of emitted electrons per second stays the same. The saturation current remains the same.

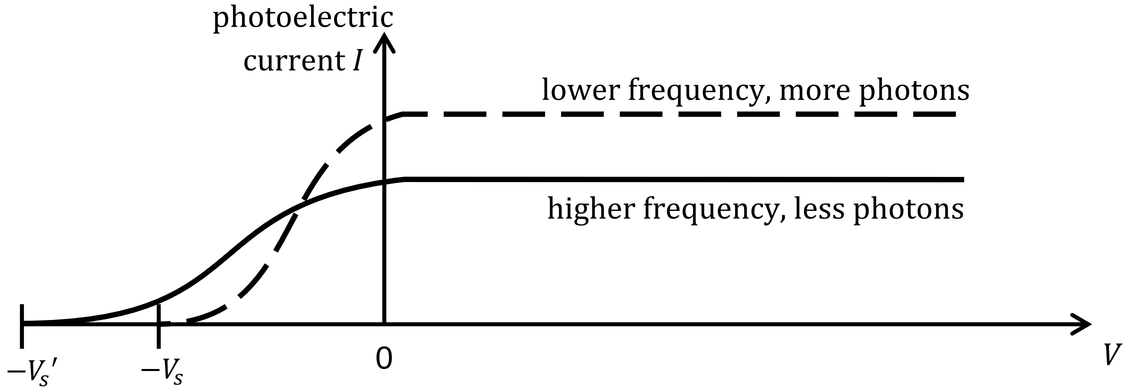


Question 1d: What if I keep the power of my light source the same, but increase the light frequency?

The total power output of the lamp is influenced the individual photon energy, as well as the number of photons per second:

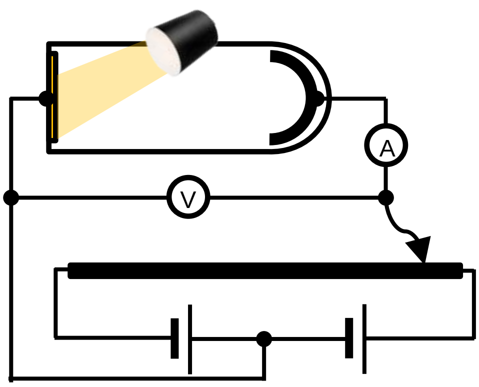
$$P_{total} = \frac{E_{total}}{t} = \left(\frac{n_{photon}}{t}\right) hf$$

If power is kept the same, the number of photons output per second will drop but the individual photon energy will increase.



Question 2: How does the stopping potential change when I increase the light frequency?

When determining the stopping potential, we adjust the potential difference across the emitter and collector plates until we read zero photoelectric current on the ammeter.

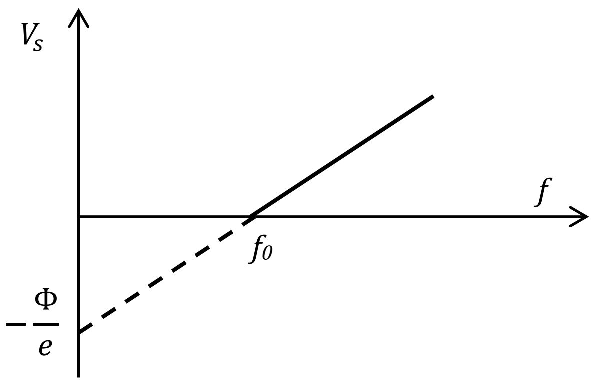


From the photoelectric equation,

$$hf = \Phi + KE_{max}$$

$$hf = \Phi + eV_s$$

$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$



The gradient of the straight line graph obtained is $\frac{h}{e}$, a constant.

The y-intercept is the work function in units of eV.

Since there is no photoelectric effect until the threshold frequency f_0 is reached, the x-intercept reflects the value of f_0.

The following table shows some of the work functions, in eV, of metals. Since photoemission occurs from the surface, do note that the values listed are of clean metal surfaces. The presence of oxidation may alter the amount of energy required to remove least tightly-bound electrons from the surfaces.

Metal	Work Function (eV)
Aluminium	4.08
Calcium	2.9
Cesium	2.1
Copper	5.0
Gold	5.1
Iron	4.5

Metal	Work Function (eV)
Magnesium	3.68
Potassium	2.3
Platinum	6.35
Silver	4.73
Sodium	2.28
Zinc	4.3

Example 15

In a photoelectric experiment, a metal with work function 1.8 eV is irradiated with light of 450 nm.

(i) Determine the stopping potential.

(ii) The metal is then replaced with another of a higher work function. Sketch the graph for the replacement metal in the diagram provided.

Solution:

(i)

From the photoelectric equation:

$$hf = \Phi + KE_{max}$$

$$\frac{hc}{\lambda} = \Phi + eV_s$$

$$V_s = \frac{hc}{\lambda e} - \frac{\Phi}{e}$$

$$= \frac{(6.63 \times 10^{-34})(3 \times 10^{8})}{(450 \times 10^{-9})(1.6 \times 10^{-19})} - 1.8$$

$$= 0.96 \text{ V}$$

(ii)

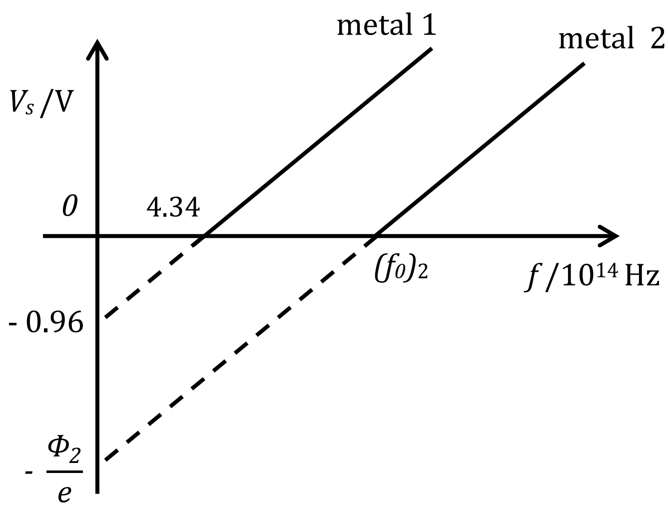
Finding threshold frequency for metal 1:

$$\Phi = hf_0$$

$$f_0 = \frac{\Phi}{h}$$

$$= \frac{(1.8)(1.6 \times 10^{-19})}{6.63 \times 10^{-34}}$$

$$= 4.3 \times 10^{14} \text{ Hz}$$



NB: Gradient is the same as $\frac{h}{e}$ is constant

18a.9 Summary

Here we summarise the ideas presented thus far:

18a.9.1 The Photoelectric Experiment

Acronym	Brief outline
E	**Emission** Photoemission is dependent on the frequency of electromagnetic radiation, and not the intensity. There is a minimum frequency, below which no electrons are emitted.
M	**Maximum** KE of photoelectrons The KE_{max} of emitted electrons is dependent on the frequency of the light and not intensity. The KE_{max} is determined by having the most energetic / highest speed electrons do work against electric field set up between emitter and collector.
I	**Instantaneous** Photoemission is instant once electromagnetic radiation of high enough frequency falls on the metal surface, regardless of intensity. No photoemission is observed even after metal surface is exposed to low-frequency, high-intensity light for extended periods of time.

18a.9.2 The Equations

Ideas	Equation
Light (EM radiation) can be particulate in nature	$E_{photon} = hf$
Shown by photoelectric effect	$hf = \Phi + KE_{max}$
Work function is indicated by threshold frequency KE_{max} is determined having photoelectrons do work against electric field generated by a stopping potential	$hf = hf_0 + eV_s$
The electrons emitted have a range of speed, up to a maximum of KE_{max}	$KE_{max} = \frac{1}{2}mv_{max}^2 = eV_s$

18a.10 Ending Notes

Albert Einstein's work in the photoelectric effect won him a Nobel Prize in Physics in 1921. This was the pre-cursor to wave-particle duality and ultimately, the development of Quantum Mechanics.

In the next topic, we shall visit the converse where electrons can be regarded as a *wave* instead of as being a particle, by showing that electron beams can be made to undergo superposition and interference, phenomena which are deem unique to waves.

A glimpse into Quantum Physics can then be made against such a backdrop of the dual nature of matter.

18a.11 Tutorial Questions

T1 A clean plate, made of metal with a work function energy of 2.36 eV, is illuminated with ultra-violet light of wavelength 370 nm. Find, in eV, the maximum energy of the emitted photoelectrons.

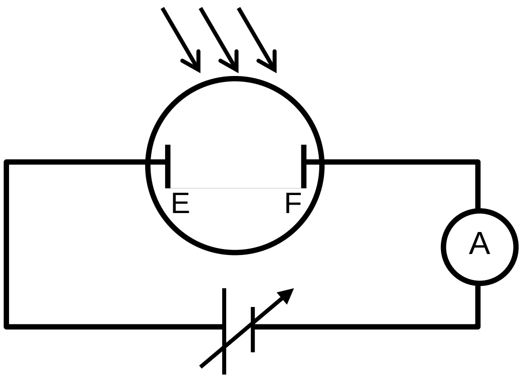
T2 When electromagnetic radiation of frequency f irradiates a metal surface, electrons are emitted and the measured stopping potential is V_S. The frequency of the incident radiation is doubled to $2f$. What change occurs in the stopping potential?

A The stopping potential increases to more than $2V_S$.
B The stopping potential increases to $2V_S$.
C The stopping potential increases to less than $2V_S$.
D The stopping potential remains at V_S.

T3 A laser emits light of power P. The light consists of photons of frequency f. The Planck constant is h and the speed of light is c. How many of these photons are contained in one metre length of the laser beam?

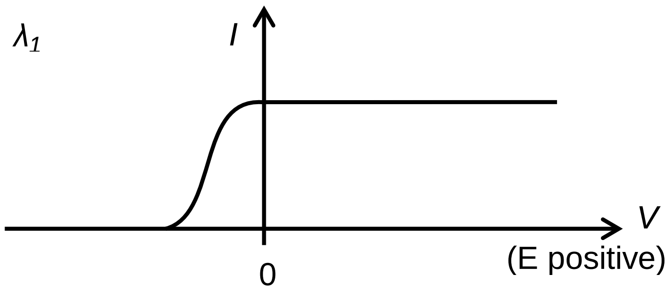
A $\frac{P}{c}$ **B** $\frac{P}{hf}$ **C** $\frac{Pc}{hf}$ **D** $\frac{P}{chf}$

T4 The diagram shows a circuit used for photoelectric emission experiments.

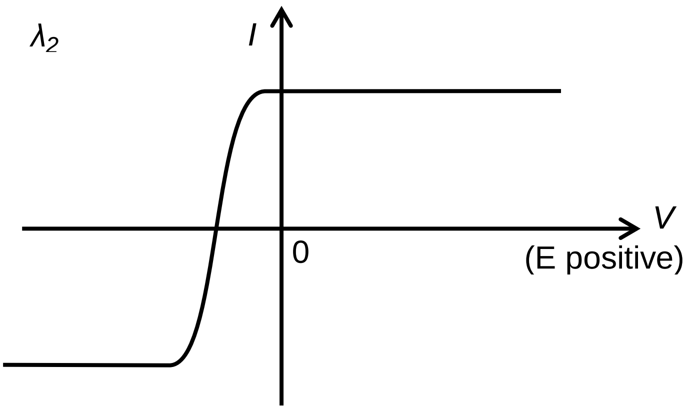


The two electrodes E and F are made of different metals. The work function of electrode E is Φ_E and the work function of electrode F is Φ_F. The current-voltage (I-V) characteristics are obtained when both electrodes are illuminated with monochromatic light.

When the wavelength of the light is λ_1, the I-V characteristic is as shown:



When the wavelength of the light is λ_2, the I-V characteristic is as shown:



Which option relates the magnitudes of the wavelengths and that of the work functions?

	wavelength	work function
A	λ_1 is less than λ_2	Φ_E is less than Φ_F
B	λ_1 is less than λ_2	Φ_E is greater than Φ_F
C	λ_1 is greater than λ_2	Φ_E is less than Φ_F
D	λ_1 is greater than λ_2	Φ_E is greater than Φ_F

T5 Light of wavelength 3.82 x 10^{-7} m is incident on a substance and electrons are emitted with a maximum speed of 6.87 x 10^{5} ms^{-1}. Calculate the work function energy of the substance.

T6 When light of wavelength 350 nm falls on a potassium surface, electrons are emitted that have a maximum kinetic energy of 1.31 eV. Determine

(a) the work function of potassium,
(b) the threshold wavelength,
(c) the threshold frequency.

T7 (a) Define threshold frequency. [1]

(b) In an experiment on photoelectric effect, a beam of light is used and the voltage across the electrodes required to stop the photocurrent is 1.5 V.

(i) Determine the maximum speed of the photoelectrons. [3]

(ii) Sketch a graph to illustrate how the photocurrent varies with the voltage across the electrodes and label it (ii). [3]

(iii) With reference to photoelectrons, explain the significance of the sloping section of your graph for negative values of potential difference. [1]

(iv) The metal is replaced by another with a greater work function. Add a new line to the graph above to indicate how the photocurrent now varies with the voltage and label it (iv). [1]

(v) With this new metal, the intensity of the light is now doubled. Add another line to the graph to indicate how the photocurrent now varies with the voltage and label it (v).

T8 (a) In a photoelectric effect demonstration, photoelectrons are being produced at a rate of 2.7 x 10^{13} per second.

Calculate the current giving this rate of production of photoelectrons.

(b) State, with a reason, what modifications to the apparatus of the demonstration would be required, if separately, to increase

(i) the energy of the photoelectron,
(ii) the rate of production of photoelectrons.

T9 (a) State an equation to express the principle of energy conservation as applied to the photoelectric effect. Explain the meaning of all the terms that you use.

(b) With reference to the photoelectric effect experiment, sketch a graph to show the variation of the stopping potential with the frequency of radiation.

(c) Explain the change in the graph if

(i) the metal surface is replaced by one with a greater work function;

(ii) the intensity of light increases.

T10 In a photoelectric emission experiment, ultraviolet radiation of wavelength 254 nm and intensity 210 Wm^{-2}, was incident on a silver surface in an evacuated tube, so that an area of 12 mm^2 was illuminated. A photocurrent of 4.8×10^{-10} A was collected at an adjacent electrode.

(a) What was the rate of incidence of photons on the silver surface?

(b) What was the rate of emission of electrons?

(c) The photoelectric quantum yield is defined as the ratio of

$$\frac{\text{number of photoelectrons emitted per second}}{\text{number of photons incident per second}}$$

(i) Find the quantum yield of this silver surface at wavelength of 254 nm.

(ii) Give two reasons why this value might be expected to be much less than one.

(d) When the experiment was repeated with the radiation of wavelength 313 nm, no photoelectron was emitted. Explain this observation.